# *VGP352 – Week 4*

- Agenda:
  - Quiz #1
  - BRDFs, part 1
    - Common ideas and terminology
    - Micro-facet based BRDFs
    - Cook-Torrance BRDF

# BRDF

- Bi-directional reflectance distribution function
  - Notation is $f(\omega_o, \omega_i)$

    "…describes the ratio of reflected radiance exiting from a surface in a particular direction (defined by the vector $\omega_o$) to the irradiance incident on the surface from direction $\omega_i$ over a particular waveband."

# BRDF

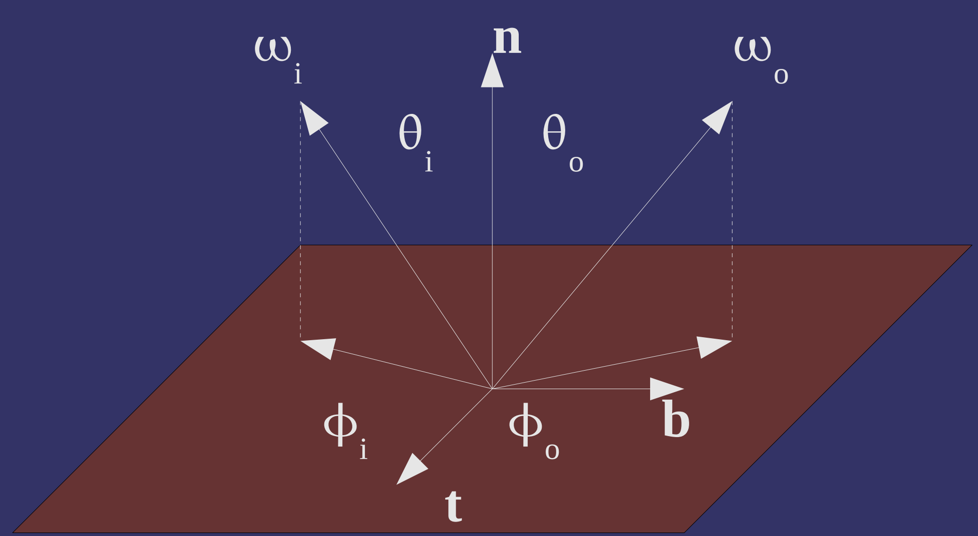
- In English...
  - Given an arbitrary input direction, $\omega_i$, and an arbitrary output direction, $\omega_o$, we can calculate the ratio of energy (light) transferred from $\omega_i$ to $\omega_o$
- What does this tell us?

# BRDF

- In English...
  - Given an arbitrary input direction, $\omega_i$, and an arbitrary output direction, $\omega_o$, we can calculate the ratio of energy (light) transferred from $\omega_i$ to $\omega_o$
- What does this tell us?
  - If we know where the light is coming from, we can calculate how much of the light is reflected in any direction
  - If we know a light reflection direction (i.e., viewing direction) we can calculate the contribution of every possible light input direction

# BRDF

- ω consists of the two angles:
  - θ is the elevation angle, and it is measured relative to the surface normal
  - φ is the azimuth angle, and it is measured relative to the surface tangent



# *BRDFs for Lighting*

- ω is a *solid angle*

  "The solid angle, Ω, is the angle in three-dimensional space that an object subtends at a point. It is a measure of how big that object appears to an observer looking from that point."[1]

  – Each ω is a direction and a "slice" from the volume of the hemisphere around the point in question

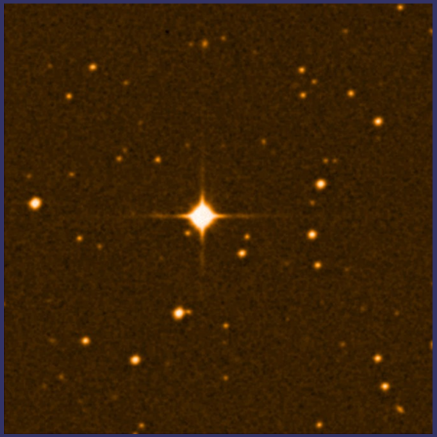
[1] From http://en.wikipedia.org/wiki/Solid_angle

# *BRDFs for Lighting*

- Why is it significant that $\omega$ is a solid angle?

# BRDFs for Lighting

- Why is it significant that ω is a solid angle?
  - The size of a light observed by the receiver matters!
    - At 20.3 light years, Earth gets no illumination from Glisse 581 → it looks tiny and has an infinitesimal solid angle
    - At ~2M miles, the planet orbiting Glisse 581 gets lots of illumination from it → it looks big and a bigger solid angle
      - 0.00000024° vs. 1.28°



[1] Image from http://en.wikipedia.org/wiki/Gliese_581

# *BRDFs for Lighting*

- The amount of light reflected from a particular input vector to a particular output vector:

$$L(\omega_o) = f(\omega_o, \omega_i) L(\omega_i) \cos\theta_i$$

Outgoing light intensity

A.k.a $\mathbf{n} \cdot \mathbf{l}$

Incoming light intensity

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# BRDFs for Lighting

- Note the similarities with our existing lighting mode:

$$L(\omega_o) = f(\omega_o, \omega_i)\, L(\omega_i) \cos\theta_i$$

$$\mathrm{L_{spec}}(\mathbf{v}) = \left(\frac{s+8}{8\pi}(\mathbf{n}\cdot\mathbf{h})^s \mathrm{c_{spec}}\right) \otimes \left(\mathbf{E_L}(\mathbf{n}\cdot\mathbf{l})\right)$$

# *BRDFs for Lighting*

- What if we want to calculate the amount light reflected to a particular output vector from *all possible* input vectors?

# BRDFs for Lighting

- What if we want to calculate the amount light reflected to a particular output vector from *all possible* input vectors?

$$L(\omega_o) = \int_\Omega f(\omega_o, \omega_i) L(\omega_i) \cos\theta_i \, d\omega_i$$

  - Integration over a solid angle works just like any other integration
  - This integral is over the hemisphere above the point
    - This is a solid angle of 2π
  - Most BRDFs will contain a 1/π factor because of this

# BRDF Properties

- Physically based BRDFs have two important properties:
  - Helmoltz reciprocity:

$$f(\omega_i, \omega_o) = f(\omega_o, \omega_i)$$

    - Also called Helmoltz Stereopsis
    - This is the “bi-directional” part of BRDF

# BRDF Properties

- Physically based BRDFs have two important properties:
  - Conservation of energy:

$$\forall\, \omega_i,\ \int_{\Omega} f(\omega_i, \omega_o) \cos\theta_o \, d\omega_o \leq 1$$

  - In other words, the output energy is less than or equal to the input energy.
  - The magic (8+s)/8 term in the modified Blinn-Phong equation conserves energy.

# *Where do BRDFs come from?*

- Measured BRDFs
  - Measure every possible output from every possible output
  - Oregon BRDF Library (and others) have data captured from these instruments available

# *Measured BRDFs*

Image from http://www.merl.com/projects/facescanning/

# Measured BRDFs

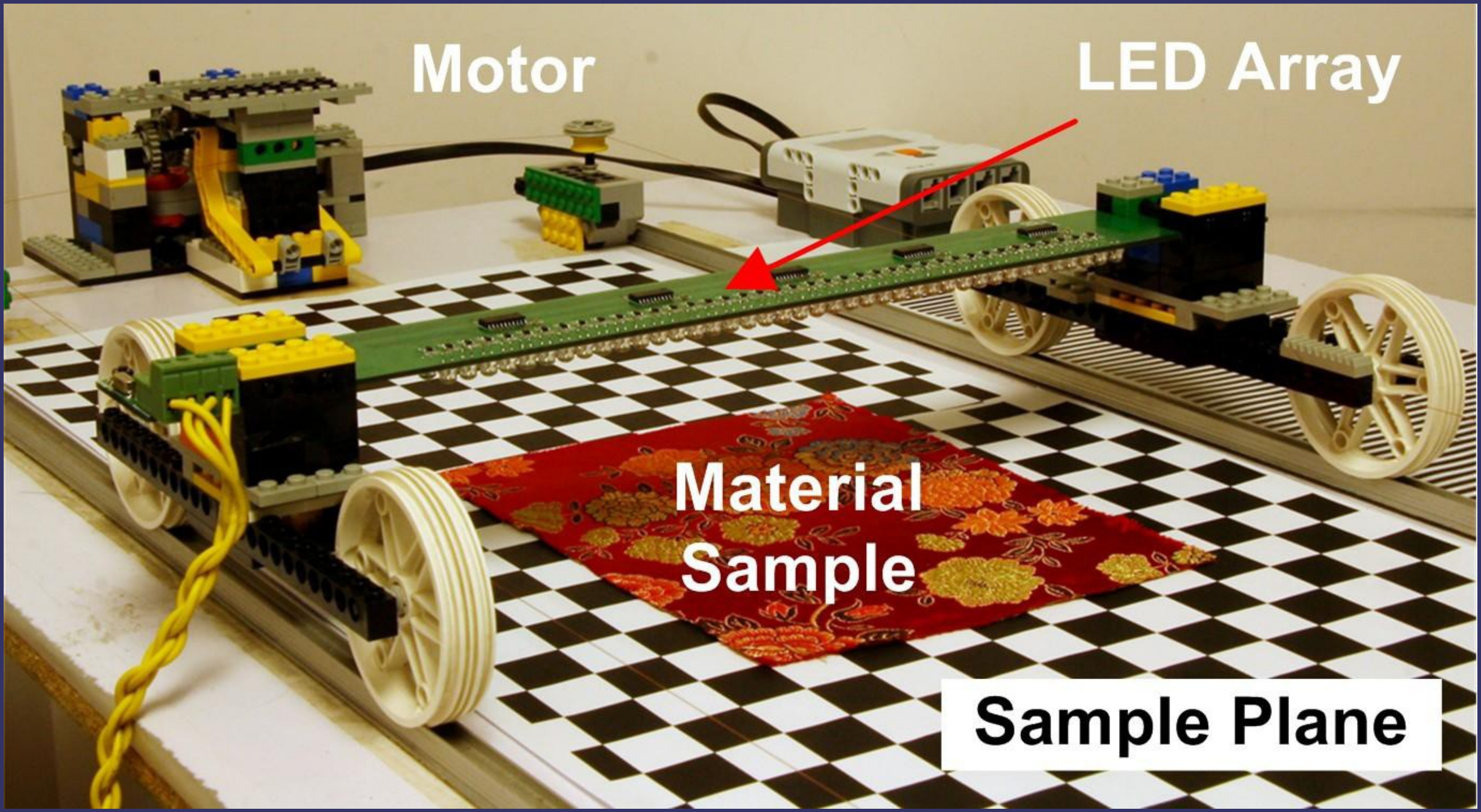


Image from http://www.shuangz.com/projects/aniso/

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# References

Wang, J., Zhao, S., Tong, X., Snyder, J., and Guo, B. 2008. Modeling anisotropic surface reflectance with example-based microfacet synthesis. In *ACM SIGGRAPH 2008 Papers* (Los Angeles, California, August 11 - 15, 2008). SIGGRAPH '08. ACM, New York, NY, 1-9. http://www.shuangz.com/projects/aniso/

McGuire, An Inexpensive Light Stage Dome. *Journal of Graphics, GPU, and Game Tools*, 2009.

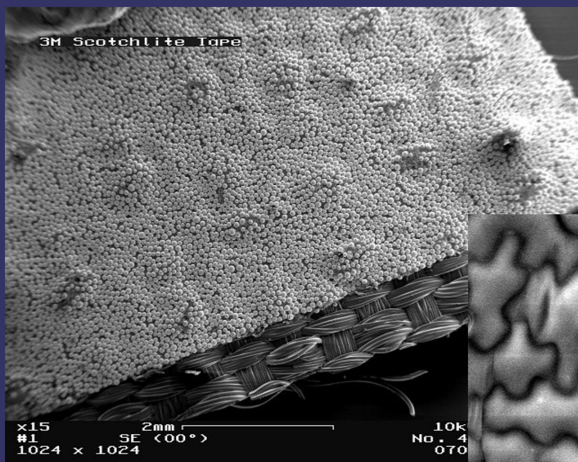
Sample BRDF data sets:

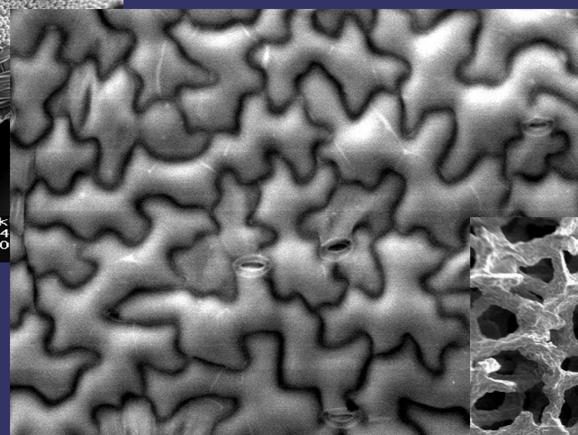
- http://www.graphics.cornell.edu/online/measurements/reflectance/index.html
- http://www1.cs.columbia.edu/CAVE//software/curet/
- http://math.nist.gov/~FHunt/appearance/obl.html

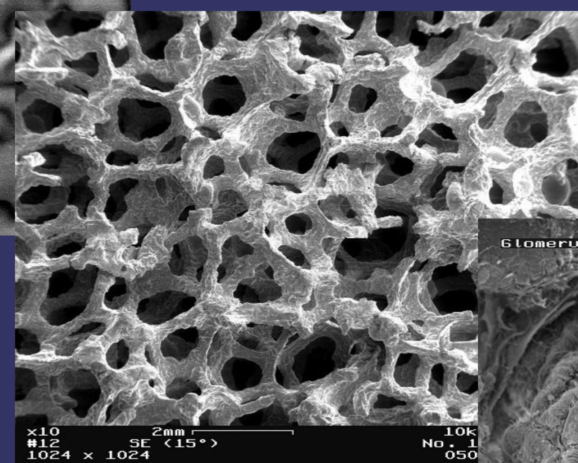
# *Where do BRDFs come from?*

- Analytical BRDFs
  - Mathematical models used to reproduce observed behavior
  - May be derived from simplified measured data

# *Micro-Facets*







Images from
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•http://commons.wikimedia.org/wiki/File:Reflective_leg_band_in_Scanning_Electron_Microscope,_15x.GIF
•http://commons.wikimedia.org/wiki/File:Metal_Foam_in_Scanning_Electron_Microscope,_magnification_10x.GIF
•http://commons.wikimedia.org/wiki/File:Glomerulum_of_mouse_kidney_in_Scanning_Electron_Microscope,_magnification_1,

# Micro-Facets

- Surfaces are made of numerous infinitesimal subsurfaces (aka micro-facets)
  - Each micro-facet acts as a perfect mirror
  - Micro-facet normals are randomly distributed according to some distribution function $p(\mathbf{h})$
  - Micro-facets can obscure other micro-facets both from the light and from the viewer

# *Micro-Facets*

- Implications:
  - Light is only reflected along the ideal reflection vector of each micro-facet
  - Distribution of the normals of these micro-facet determines how specular the surface appears
  - Amount of internal occlusion limits reflection

# *Micro-Facets*

- Implications:
  - Light is only reflected along the ideal reflection vector of each micro-facet
  - Distribution of the normals of these micro-facet determines how specular the surface appears
  - Amount of internal occlusion limits reflection

Determining the number of facets with $\mathbf{n} = \mathbf{h}$ that are visible to $\mathbf{v}$ and $\mathbf{l}$ is enough to determine the BRDF.

# *Micro-Facets*

- BRDF is determined by:
  - Fresnel term
  - Fraction of micro-facets with $\mathbf{n} = \mathbf{h}$
  - Fraction of micro-facets visible to both $\mathbf{l}$ and $\mathbf{v}$
    - Non-visible to $\mathbf{l}$ is often called “shadowing”
    - Non-visible to $\mathbf{v}$ is often called “masking”
    - Both can just be called “occlusion”

# *Distribution of Micro-Facet Normals*

- Micro-facet normals are random, but follow some distribution function
  - Given **n**, determine the fraction of micro-facet normals that point towards **h**
    - Sometimes called the *normal distribution function* (NDF)
  - Can use arbitrary function to calculate this probability
    - May be convenient to encode this in a texture
  - Gaussian or *standard normal distribution* function seems like a good choice
    - The more different the **h** is from **n**, the lower the probability

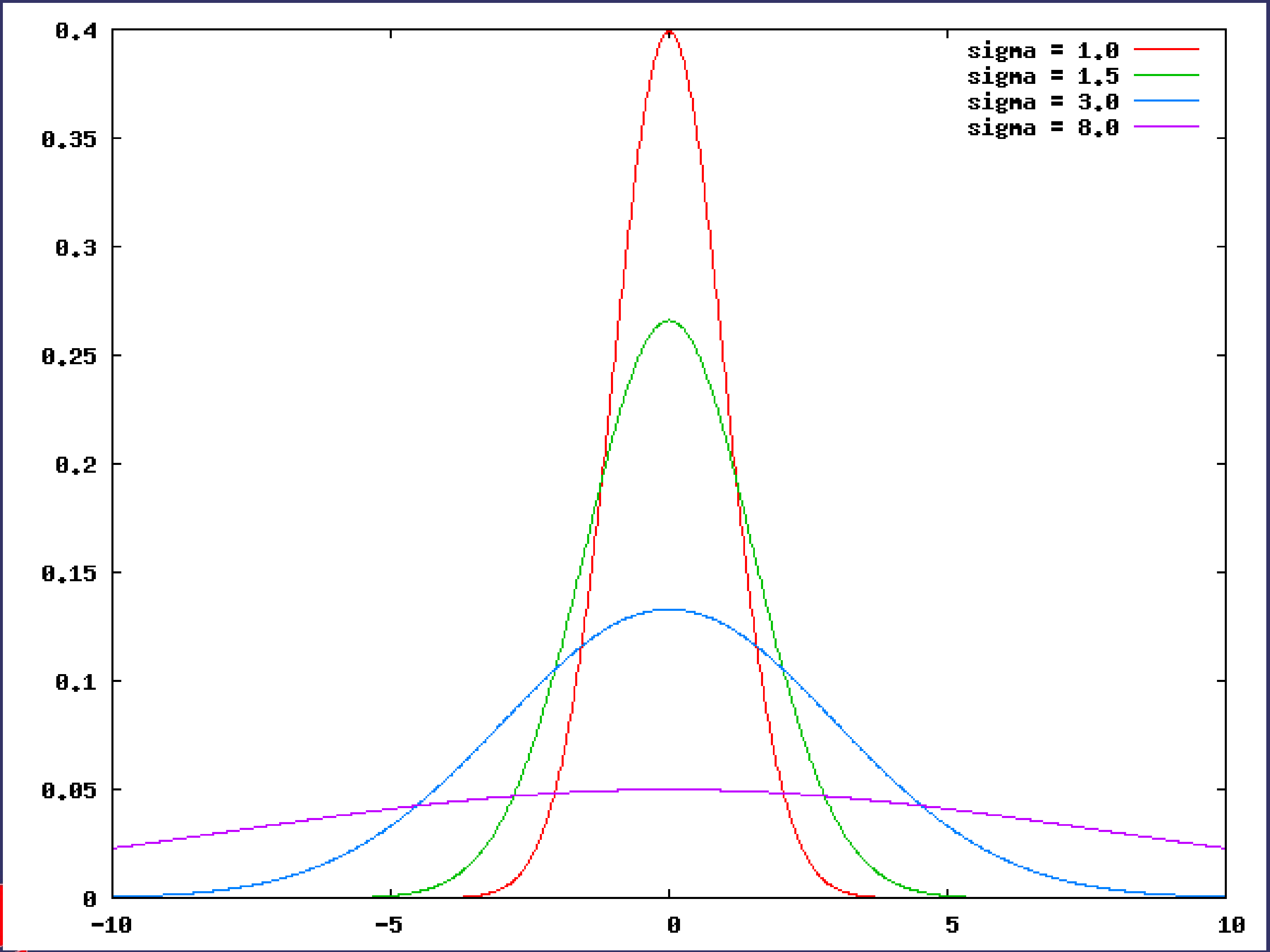
# Distribution of Micro-Facet Normals

- Gaussian distribution:

$$P(\theta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{\theta^2}{2\sigma^2}\right)}$$

$\sigma$ is the standard deviation

# Distribution of Micro-Facet Normals



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- Looking at the graph, why is this distribution unsuitable?

# *Distribution of Micro-Facet Normals*

- Gaussian distribution:

$$P(\theta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{\theta^2}{2\sigma^2}\right)}$$

$\sigma$ is the standard deviation

- Looking at the graph, why is this distribution unsuitable?
  - As $\sigma$ increases, the effective range increases to $\infty$
    - Cook-Torrance paper includes a scaling factor to counteract this, but it's fidgety.
  - Distribution is based on $\theta$, but we only know $\cos(\theta)$

# Distribution of Micro-Facet Normals

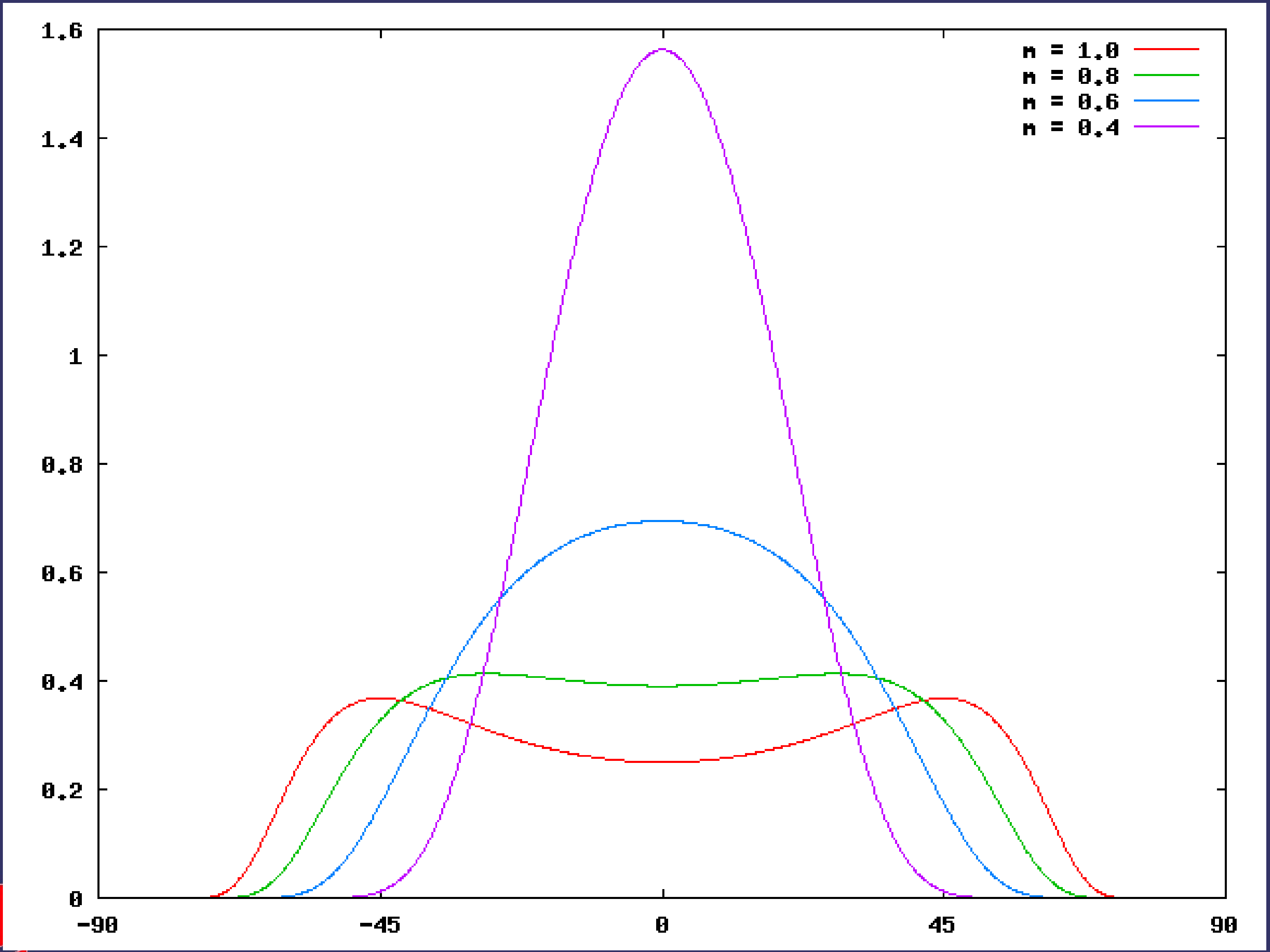
- Beckmann distribution:

$$P(\theta) = \frac{1}{4 m^2 \cos^4 \theta} e^{-\left(\frac{\tan^2 \theta}{m^2}\right)}$$

$m$ is average slope of the surface micro-facets

- Physically based model of rough surfaces
  – Based on Petr Beckmann's research in the early 60s
- All calculations are based on $\cos(\theta)$!
  – Remember: $\tan^2(\theta)$ is (1 - $\cos^2(\theta)$) / $\cos^2(\theta)$

# Distribution of Micro-Facet Normals

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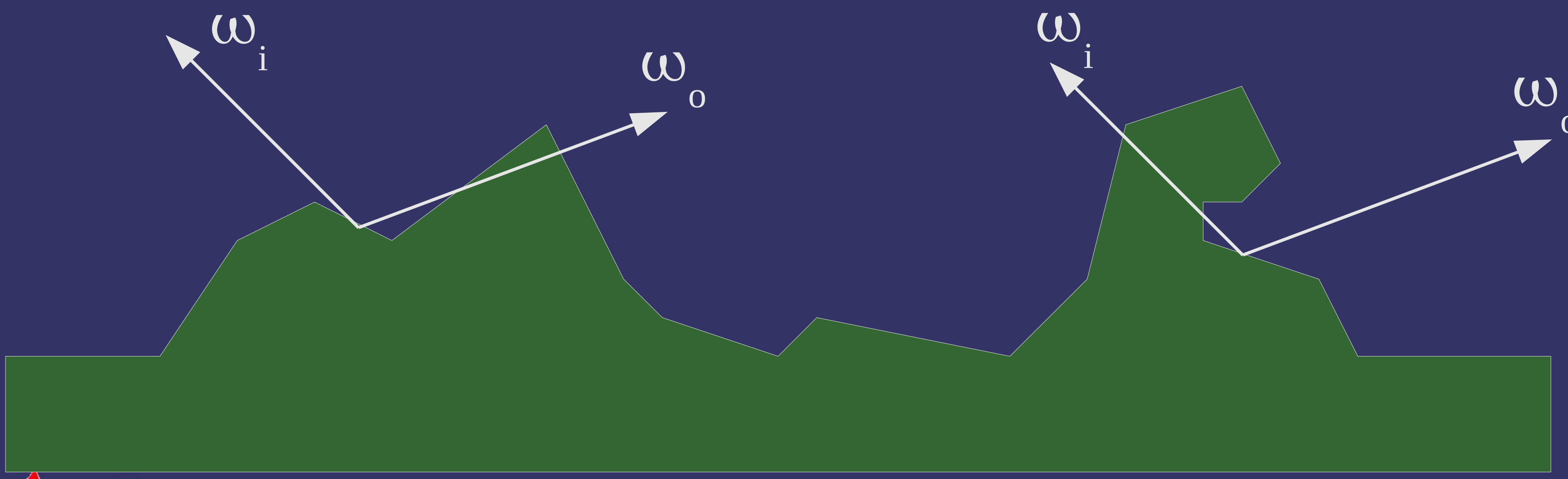
# Micro-Facet Occlusion

- A facet only contributes if it is visible to both $\mathbf{v}$ and $\mathbf{l}$
  - Need to know the probability of a facet being visible



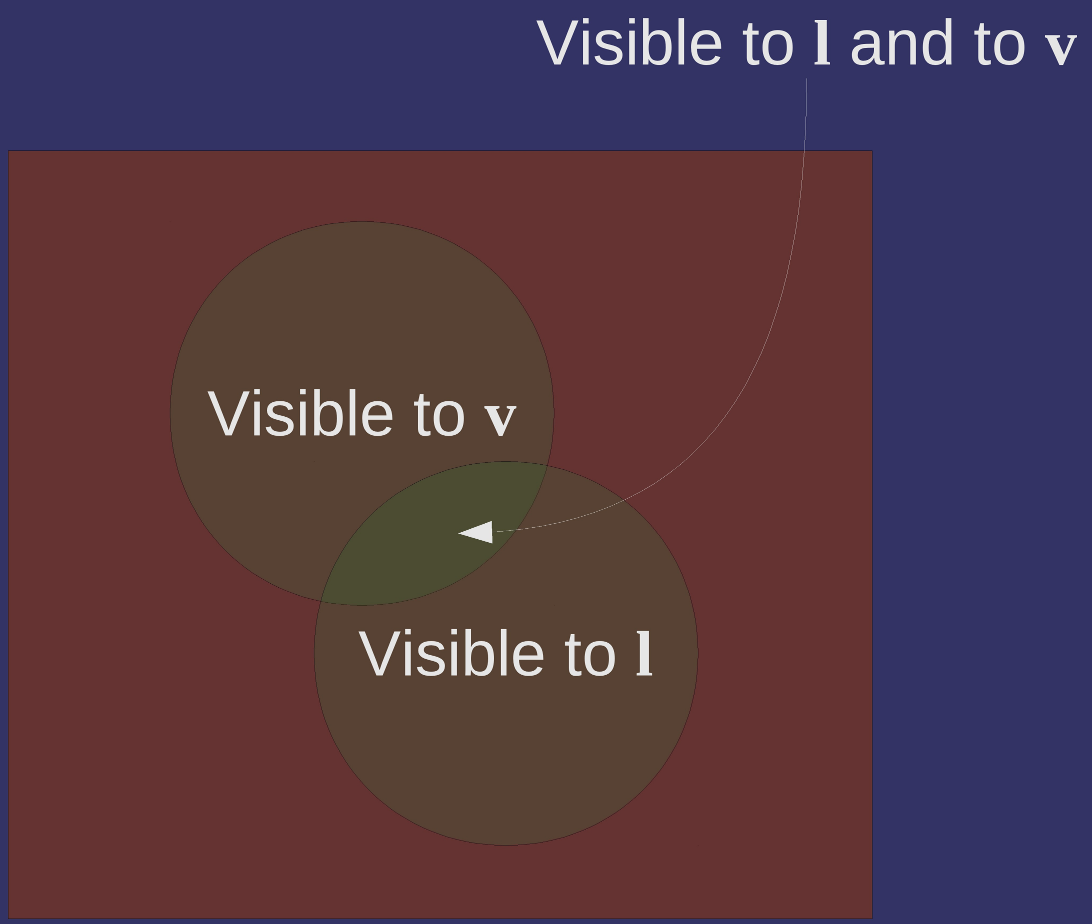
# *Micro-Facet Occlusion*

- Determine the probability of a facet being visible to the light and to the viewer
  - Use one probability function, $P_{vis}(\theta)$, for the probability of visibility to either $\mathbf{l}$ or $\mathbf{v}$
  - Assume that visibility and orientation are uncorrelated

# Micro-Facet Occlusion

- Given $P_{vis}(\theta_{\mathbf{v}})$ and $P_{vis}(\theta_{\mathbf{l}})$, what is $P_{vis}(\theta_{\mathbf{v}} \cap \theta_{\mathbf{l}})$?

# *Micro-Facet Occlusion*

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# *Micro-Facet Occlusion*

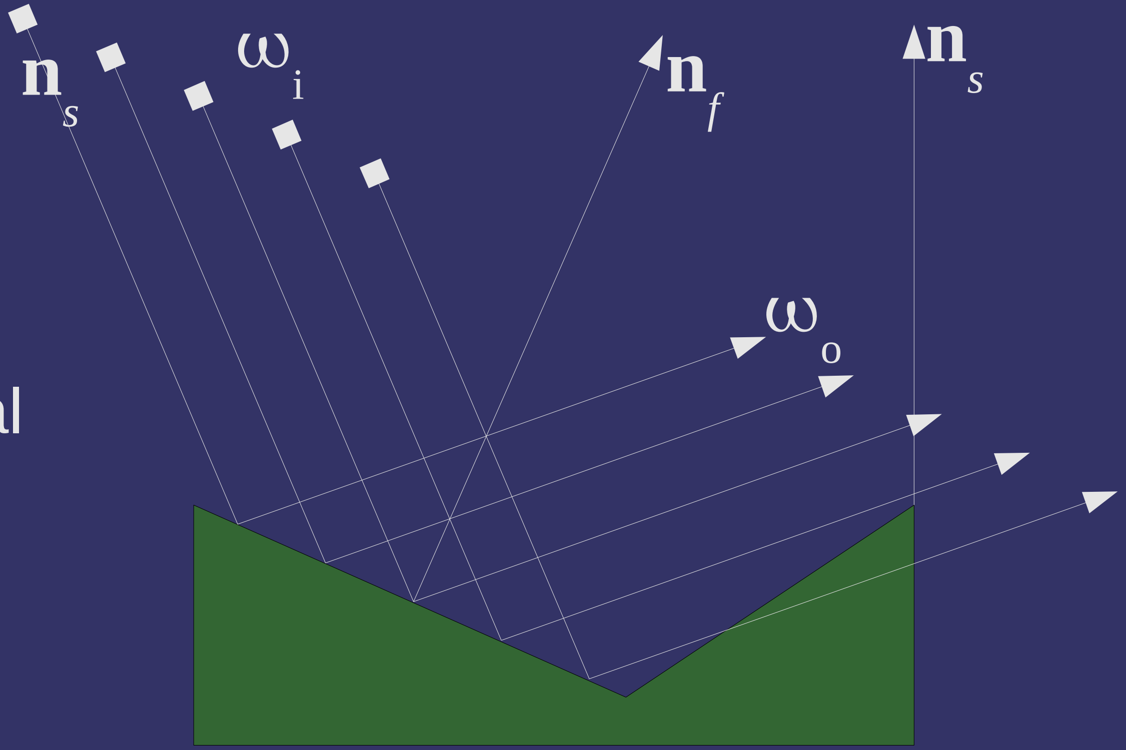
- Given $P_{vis}(\theta_{\mathbf{v}})$ and $P_{vis}(\theta_{\mathbf{l}})$, what is $P_{vis}(\theta_{\mathbf{v}} \cap \theta_{\mathbf{l}})$?
  - Calculating $P(A \cap B)$ from $P(A)$ and $P(B)$ is hard, in general.
  - $P_{vis}(\theta_{\mathbf{v}}) \times P_{vis}(\theta_{\mathbf{l}}) < P_{vis}(\theta_{\mathbf{v}} \cap \theta_{\mathbf{l}})$
    - $P(A)P(B) = P(A \cap B) \leftrightarrow A$ and $B$ are independent
    - Visibility to the light and viewer are ***not*** independent
      - Example: Put the light and viewer at the same location
  - Different BRDFs use different methods

# Micro-Facet Occlusion

- Simplifying assumption:
  - Model the surface as a bunch of parallel grooves
  - Grooves *magically* keep the same orientation relative to the viewer no matter how the surface is oriented

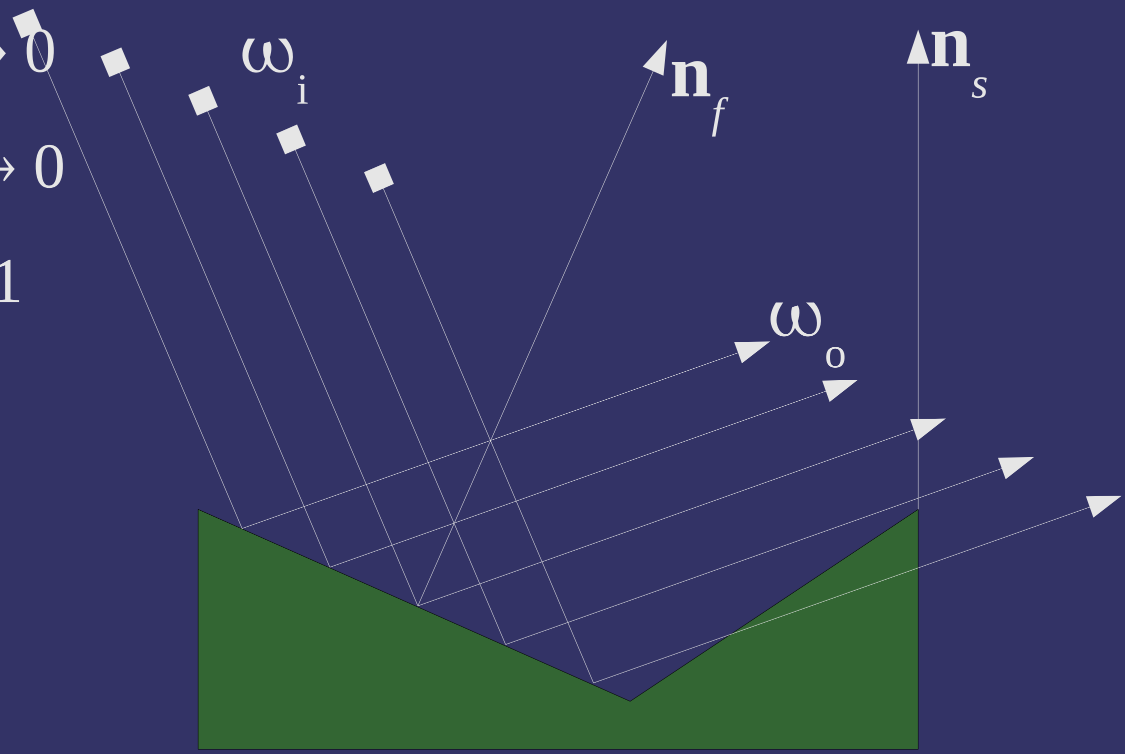
# *Micro-Facet Occlusion*

- How do we estimate $P_{vis}(\theta)$?
  - Clearly $\omega_i$, $\omega_o$, $\mathbf{n}_f$, and $\mathbf{n}_s$ are involved
    - $\mathbf{n}_f$ is the facet normal
    - $\mathbf{n}_s$ is the surface normal



# Micro-Facet Occlusion

- Observations:
  - Occlusion increases as:
    - $\angle \mathbf{n}_f \mathbf{n}_s \to 90^\circ \Leftrightarrow (\mathbf{n}_f \cdot \mathbf{n}_s) \to 0$
    - $\angle \omega \mathbf{n}_s \to 90^\circ \Leftrightarrow (\omega \cdot \mathbf{n}_s) \to 0$
    - $\angle \omega \mathbf{n}_f \to 0^\circ \Leftrightarrow (\omega \cdot \mathbf{n}_f) \to 1$

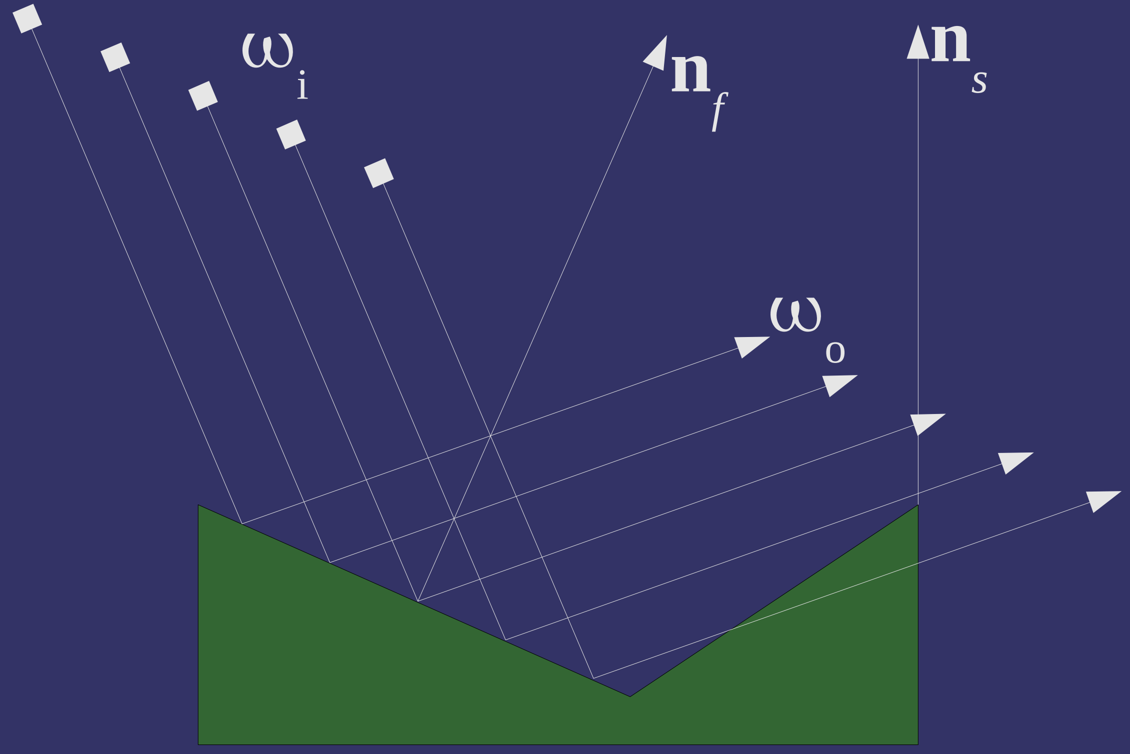


# Micro-Facet Occlusion

- Cook-Torrance uses:

$$P_v(\theta) = \frac{2(\mathbf{n}_s \cdot \mathbf{n}_f)(\mathbf{n}_s \cdot \omega)}{\omega \cdot \mathbf{n}_f}$$

- What other vector is equivalent to $\mathbf{n}_f$?



# Micro-Facet Occlusion

- Cook-Torrance uses:

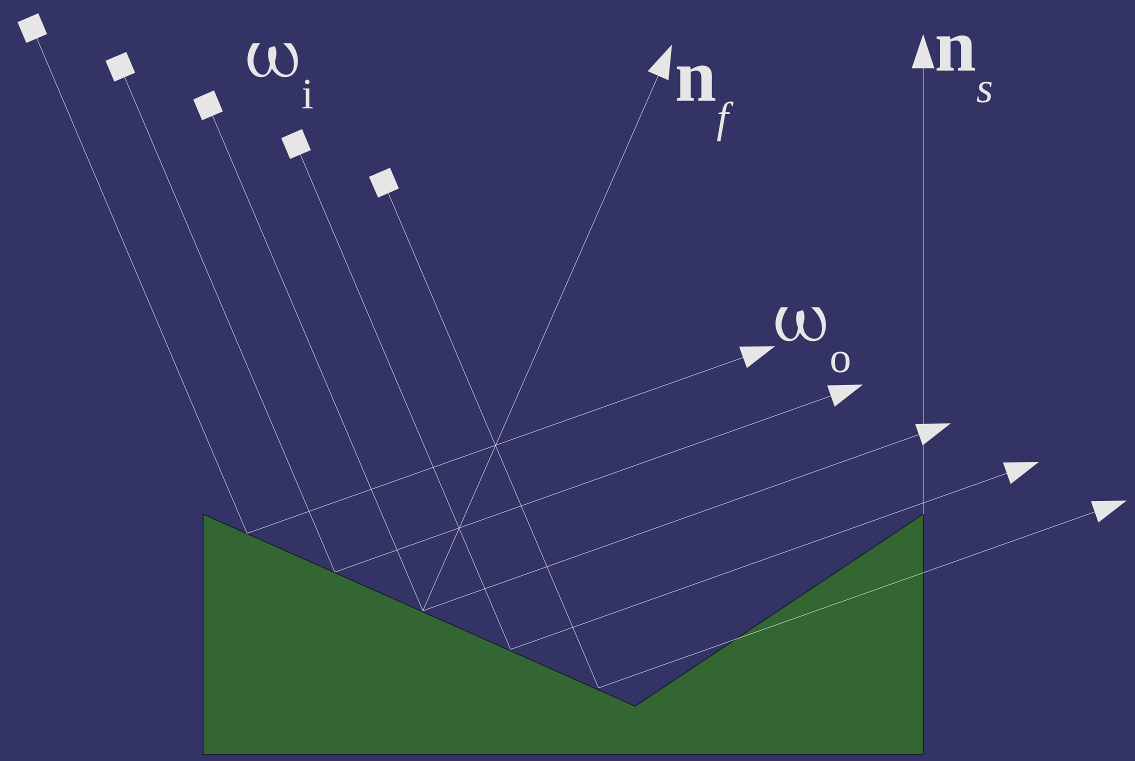
$$P_v(\theta) = \frac{2(\mathbf{n}_s \cdot \mathbf{n}_f)(\mathbf{n}_s \cdot \omega)}{\omega \cdot \mathbf{n}_f}$$

- What other vector is equivalent to $\mathbf{n}_f$?
  - By definition, $\mathbf{n}_f = \mathbf{h}$

$$G_\mathbf{v} = \frac{2(\mathbf{n} \cdot \mathbf{h})(\mathbf{n} \cdot \mathbf{v})}{\mathbf{v} \cdot \mathbf{h}}$$

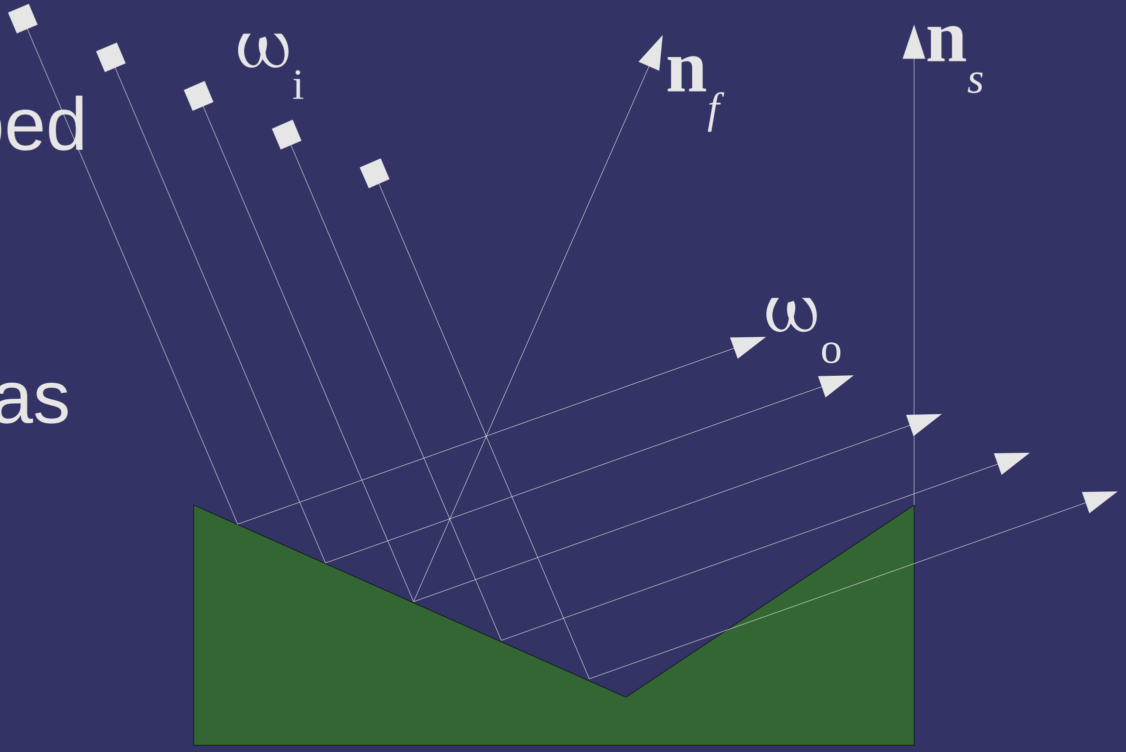
$$G_\mathbf{l} = \frac{2(\mathbf{n} \cdot \mathbf{h})(\mathbf{n} \cdot \mathbf{l})}{\mathbf{l} \cdot \mathbf{h}}$$

$$\mathbf{l} \cdot \mathbf{h} = \mathbf{v} \cdot \mathbf{h}$$



# Micro-Facet Occlusion

- This turns out to be a poor model
  - Real surfaces aren't made of long, V-shaped channels
  - The reading for next week addresses this as well



# *Cook-Torrance BRDF*

- One of the oldest BRDFs used in graphics
  - Published by Robert Cook and Ken Torrance in 1982
    - Cook was at Lucasfilm, Ltd.
    - Torrance was at Cornell
  - Based on *micro-facets*

# Cook-Torrance BRDF

$$
\begin{aligned}
f(\omega_o, \omega_i) &= \mathbf{k}_d f_d + \mathbf{k}_s f_s(\omega_o, \omega_i) \\
f_d &= 1/\pi \\
f_s(\omega_o, \omega_i) &= 1/\pi \frac{F \times D(\mathbf{n} \cdot \mathbf{h}) \times G(\mathbf{n} \cdot \omega_i, \mathbf{n} \cdot \mathbf{h}, \mathbf{n} \cdot \omega_o)}{(\mathbf{n} \cdot \omega_i)(\mathbf{n} \cdot \omega_o)}
\end{aligned}
$$

- $F$ is the Fresnel factor
- $D$ is the distribution of micro-facet normals
- $G$ is the geometry occlusion factor
- $\mathbf{h}$ is the half-vector from the Blinn-Phong lighting equation

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# Cook-Torrance Diffuse Factor

- Cook-Torrance diffuse factor:

$$f_d = 1/\pi$$

- Lambertian diffuse factor:

$$\mathbf{k}_d = \mathbf{n} \cdot \mathbf{l}$$

- Remember how the BRDF is used:

$$L(\omega_o) = f(\omega_o, \omega_i) L(\omega_i) \cos\theta_i$$

  - We just want to scale the incoming energy by the total angle and let the built in $(\mathbf{n} \cdot \omega_i)$ do the rest
  - Remember $\omega_i \cong \mathbf{l}$

# Cook-Torrance Normal Distribution

- Cook-Torrance uses the Beckmann Distribution:
  - $m$ is a parameter that controls the smoothness of the surface

$$\mathrm{D}(\mathbf{n}\cdot\mathbf{h}) = \frac{1}{4m^2(\mathbf{n}\cdot\mathbf{h})^4} e^{\left(\frac{(\mathbf{n}\cdot\mathbf{h})^2-1}{(\mathbf{n}\cdot\mathbf{h})^2 m^2}\right)}$$

# Cook-Torrance Geometry Occlusion

- Represents the decrease in light transmission caused by occlusion of the light or viewer by other micro-facets

$$G(\mathbf{n}\cdot\omega_i, \mathbf{n}\cdot\mathbf{h}, \mathbf{n}\cdot\omega_o) = \min\left(1, \frac{2(\mathbf{n}\cdot\mathbf{h})(\mathbf{n}\cdot\omega_o)}{\omega\cdot\mathbf{h}}, \frac{2(\mathbf{n}\cdot\mathbf{h})(\mathbf{n}\cdot\omega_i)}{\omega\cdot\mathbf{h}}\right)$$

- Why aren't there any subscripts on $\omega$ in the denominators?
  - Hint: $\omega_i \cong \mathbf{l}$ and $\omega_o \cong \mathbf{v}$

# Cook-Torrance Geometry Occlusion

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$$G(\mathbf{n}\cdot\omega_i, \mathbf{n}\cdot\mathbf{h}, \mathbf{n}\cdot\omega_o) = \min\left(1, \frac{2(\mathbf{n}\cdot\mathbf{h})(\mathbf{n}\cdot\omega_o)}{\omega\cdot\mathbf{h}}, \frac{2(\mathbf{n}\cdot\mathbf{h})(\mathbf{n}\cdot\omega_i)}{\omega\cdot\mathbf{h}}\right)$$

- Why aren't there any subscripts on $\omega$ in the denominators?
  - Hint: $\omega_i \cong \mathbf{l}$ and $\omega_o \cong \mathbf{v}$
  - $\mathbf{h}$ is half way between $\mathbf{v}$ and $\mathbf{l}$:

$$\angle\,\mathbf{l}\,\mathbf{h} = \angle\,\mathbf{v}\,\mathbf{h} \therefore (\mathbf{h}\cdot\omega_i) = (\mathbf{h}\cdot\omega_o)$$

# References


http://wiki.gamedev.net/index.php/D3DBook:(Lighting)_Cook-Torrance

http://en.wikipedia.org/wiki/Specular_highlight#Beckmann_distribution

Philip Dutré. “Global Illumination Compendium.” Computer Graphics, Department of Computer Science Katholieke Universiteit Leuven. 2003. http://www.cs.kuleuven.ac.be/~phil/GI/

# *Reading for Next Week*

Prepare for next week:

Gregory J. Ward. 1992. Measuring and modeling anisotropic reflection. In *Proceedings of the 19th annual conference on Computer graphics and interactive techniques* (SIGGRAPH '92), James J. Thomas (Ed.). ACM, New York, NY, USA, 265-272. http://www.cs.virginia.edu/~gfx/courses/2006/DataDriven/bib/appearance/ward92.pdf

**NOTE:** This is different than what is (was) listed in the syllabus!

# *Next week...*

- Quiz #2
- More BRDFs
  - Anisotropic reflection
    - Ward BRDF
    - Ashikhmin BRDF
  - Metals
    - How do metals “reflect” light?
    - Lafortune BRDF

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