VGP351 – Week 6

- Agenda:
  - Bounding volumes
    - Axis-aligned bounding boxes
    - Oriented bounding boxes
    - Bounding spheres
  - BV hierarchies

Bounding Volumes

- From Wikipedia:

  "...a bounding volume for a set of objects is a closed volume that completely contains the union of the objects in the set."

- Why is this useful?

Bounding Volumes

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  "...a bounding volume for a set of objects is a closed volume that completely contains the union of the objects in the set."

- Why is this useful?
  - Can represent complex geometry that would be expensive to test with an approximation that is much cheaper to test
  - Visibility, collision detection, etc.

Desirable BV Characteristics

- Inexpensive intersection test
  - BVs are used instead of source geometry to speed up *trivial rejection* (or trivial acceptance) tests

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- Inexpensive intersection test
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- Tight fitting to source geometry
  - If the BV is a poor fit, tests between BVs may result in false positives or false negatives
- Inexpensive to compute
  - If the BV is too expensive to compute, the expense of creating it may cancel the speed-up that it provides

Desirable BV Characteristics

- Easy to transform
  - If the object moves, its BV needs to move. If moving the BV is too expensive, it may cancel out the speed-up.

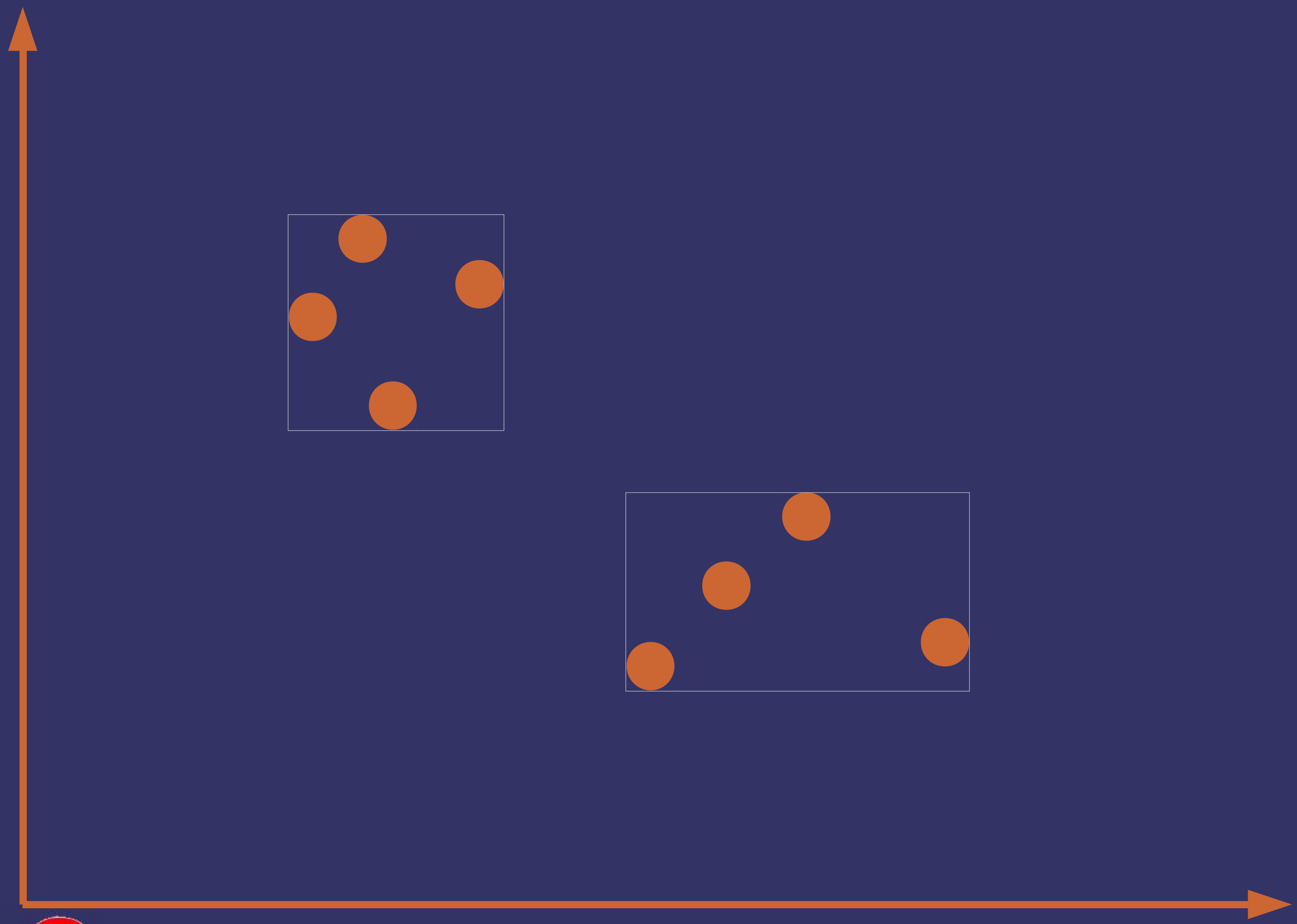
Desirable BV Characteristics

- Easy to transform
  - If the object moves, its BV needs to move. If moving the BV is too expensive, it may cancel out the speed-up.
- Inexpensive to store
  - If the BV requires too much space to store or too much time to access, it can negatively impact performance.

Axis-Aligned Bounding Box

- AABB is probably the most common bounding volume
  - Just an n-dimensional box with sides parallel to the principle axes that encloses all the points

Axis-Aligned Bounding Box



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Axis-Aligned Bounding Box

- Three common representations
  - Easy to translate between them
  - Which is used depends on the source data and the usage of the BV

Axis-Aligned Bounding Box

```
class aabb_min_max {
    // Points such that for every point P in the
    // object:
    //     (min.x <= P.x <= max.x)
    //     && (min.y <= P.y <= max.y)
    //     && (min.z <= P.z <= max.z)
    GLUvec4 min;
    GLUvec4 max;
};
```

Axis-Aligned Bounding Box

```
class aabb_min_diameter {
    // Points such that for every point P in the
    // object:
    //     (min.x <= P.x)
    //     && (min.y <= P.y)
    //     && (min.z <= P.z)
    GLUvec4 min;

    // Dimensions of the box in each direction
    GLUvec4 diameter;
};
```

Axis-Aligned Bounding Box

```
class aabb_center_radius {
    // Center of the bounding box
    GLUvec4 center;

    // Radius of the box in each direction
    GLUvec4 radius;
};
```

AABB Creation

- Trivial O(n) problem:
  - Scan all points tracking minimum and maximum value in each dimension

AAB B Update

- Translation is trivial
  - Rotation is problematic
- Three common techniques:
  - Recalcuation
  - AABB of an AABB
  - Hill climbing

AABB Update

- Recalculation:
  - Transform source data, calculate new AABB
- Advantages / disadvantages?

AABB Update

- Recalculation:
  - Transform source data, calculate new AABB
- Advantages / disadvantages?
  - Creates a tight-fitting AABB
  - $O(n)$ per transformation is probably much too slow
    - Can speed up by using only points on the convex hull

AABB Update

- Hill climbing:
  - Track the extreme points of the object
  - To update, check neighboring points for new extrema
- Advantages / disadvantages?

AAB B Update

- Hill climbing:
  - Track the extreme points of the object
  - To update, check neighboring points for new extrema
- Advantages / disadvantages?
  - Creates a tight-fitting AABB
  - Average case performance is good
    - Requires precalculation of convex hull
    - Requires data structure to store connectivity among points on hull

AABB Update

- AABB of AABB:
  - Calculate AABB of base orientation of object
  - Apply transformations to object and AABB
  - Calculate AABB of transformed AABB
- Advantages / disadvantages?

AABB Update

- AABB of AABB:
  - Calculate AABB of base orientation of object
  - Apply transformations to object and AABB
  - Calculate AABB of transformed AABB
- Advantages / disadvantages?
  - Creates a loose-fitting AABB
  - Very fast!
- This is probably the most commonly used technique

Oriented Bounding Boxes

- Arbitrarily oriented box that encloses the object
  - Can lead to much tighter bounding volume
- How would you represent an OBB?

Oriented Bounding Boxes

```
class obb_base_vectors {
    // Base point of box
    GLUvec4 base;

    // X, Y, and Z axes
    GLUvec4 axis[3];
};
```

Oriented Bounding Boxes

```
class obb_basis_radius {
    // Radius in each direction
    GLUvec4 radius;

    // Transformation to the OBB's coordinate
    // system
    GLUmat4 basis;
};
```

OBB Creation

- One common method:
  - Calculate 3D convex hull
    - One of the OBB faces must be coplanar with a face of the convex hull
  - For each face of the 3D convex hull:
    - Project points onto its plane
    - Calculate 2D convex hull
    - Use “rotating calipers” to find minimal bounding rectangle
      - This defines one face of the OBB
    - Calculate distance of farthest point from the convex hull face
  - Use the OBB with the smallest resulting volume

OBB Creation

- References:

http://cbloomrants.blogspot.com/2009/04/04-24-09-convex-hulls-and-obb.html

OBB Update

- Trivial!
  - Apply transformation to the OBB's basis matrix

Bounding Spheres

- Sphere surrounding the object
  - Ideally it's the *minimal* sphere
  - Representation is trivial
  - Update is trivial

Bounding Sphere Creation

- Generating a *good* sphere is non-trivial
  - Brute-force is $O(n^5)$
  - Statistical methods can approximate in $O(n)$
  - A recursive method can produce min. sphere in $O(n)$
    - A robust implementation is complex.
  - An iterative approach can get ~5% of min. in $O(n)$
    - Has a higher constant factor.

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We won't talk about these methods today

Bounding Sphere Creation

- Brute-force:
  - A plane is defined by 3 non-colinear points
  - A sphere is defined by 3 points on a plane and one point not on the plane
    - i.e., a tetrahedron
  - Consider the sphere defined by all combinations of 4 non-coplanar points, keep the smallest that contains all the points.

Bounding Sphere Creation

- Ritter's algorithm:
  - Given an initial guess that is too small, can find bounding sphere within 10% of minimum
  - Easy to understand and easy to implement
    - I did a version in 68000 assembly language many years ago

Bounding Sphere Creation

```
void bounding_sphere(Sphere &sphere, GLUvec4 *p, unsigned num)
{
    float r_squared = sphere.radius * sphere.radius;

    for (unsigned i = 0; i < num; i++) {
        const GLUvec4 d = p[i] – sphere.center;
        const float dist_squared = gluDot3(d, d);

        if (dist_squared > r_squared) {
            const float dist = sqrt(dist_squared);
            const float r = (sphere.radius + dist) / 2.0f;
            const float k = (r – sphere.radius) / dist;

            sphere.radius = r;
            sphere.center += d * k;
            r_squared = r * r;
        }
    }
}
```

Bounding Sphere Creation

- What's the big assumption?

Bounding Sphere Creation

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  - That we have a good way to come up with an initial sphere
    - The initial sphere must be a little bit too small
    - The better the initial sphere, the better the final sphere

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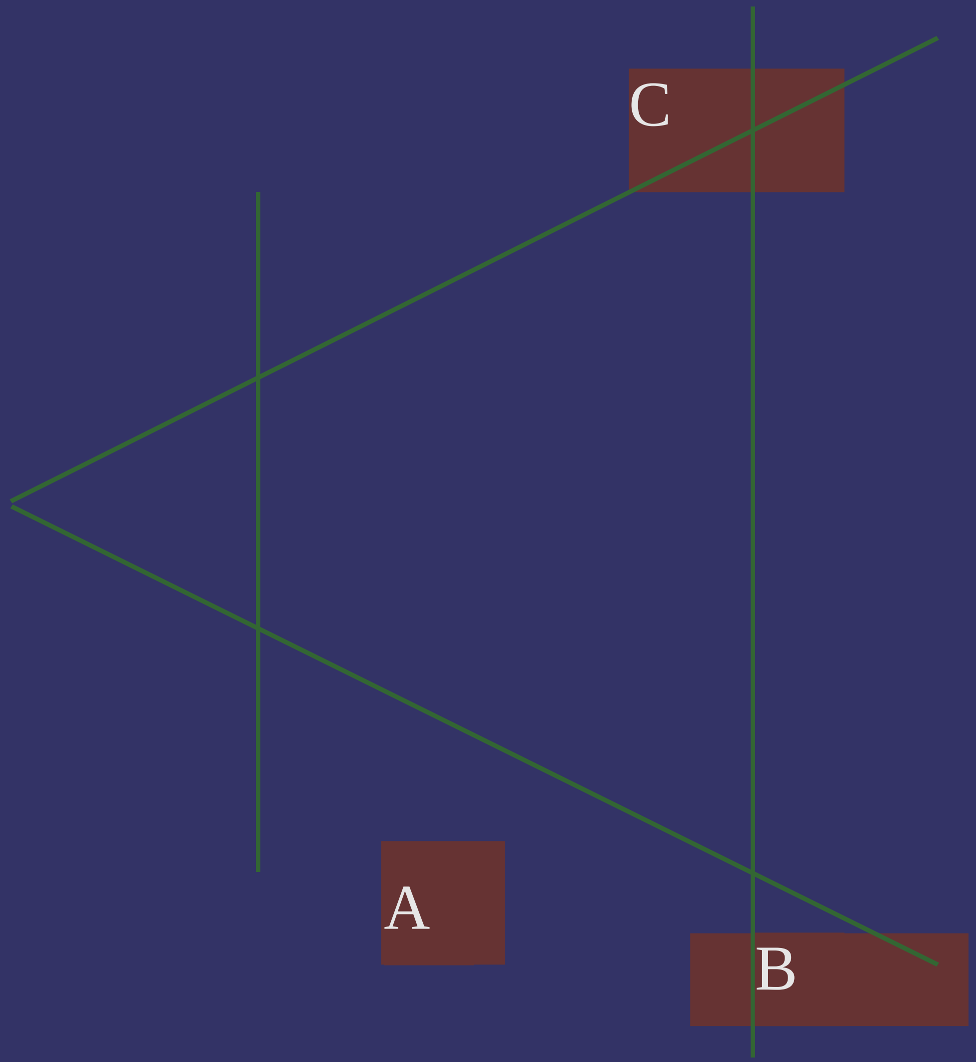
  - That we have a good way to come up with an initial sphere
    - The initial sphere must be a little bit too small
    - The better the initial sphere, the better the final sphere
- Apply the algorithm repeatedly
  - Generate a sphere from an AABB
  - Apply Ritter's algorithm
  - Shrink the output sphere
  - Apply again adding the points in random order

AABB / Frustum Intersection

- Test each corner of the box. If all corners are outside the frustum, then box is outside.



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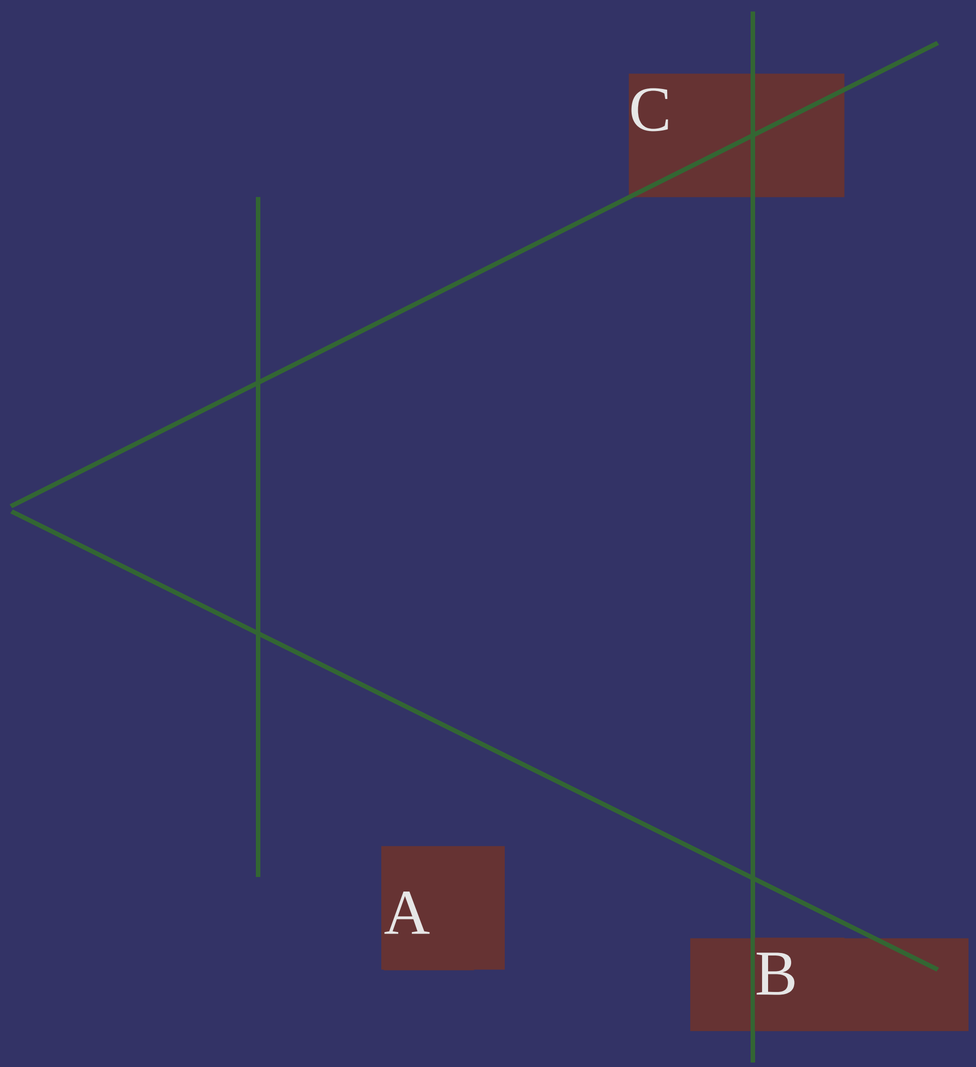
AABB / Frustum Intersection

- ~~Test each corner of the box. If all corners are outside the frustum, then box is outside.~~ Wrong!!!
- If all corners are on positive side of any one plane, then the box is outside.



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AABB / Frustum Intersection

- Can we do better than testing all 8 corners?

AABB / Frustum Intersection

- Can we do better than testing all 8 corners?
  - Pick the “most positive” point and “most negative” point relative to each plane
    - Call these the *p-vertex* and the *n-vertex*
  - Test just those points
    - If both are on the same side of the plane, then **all** of the points must be on that same side

AABB / Frustum Intersection

- Finding p-vertex and n-vertex:
  - Look at the signs of the components of the plane's normal
  - The signs determine which corner the normal points towards
    - Example: If the normal signs are { +, +, - }, then the p-vertex is { box.radius.x, box.radius.y, -box.radius.z }
    - The n-vertex is always the opposite corner

AABB / Frustum Intersection

```
int frustum_aabb(Plane *planes, Aabb &aabb)
{
    bool intersect = false;
    for (unsigned i = 0; i < 6; i++) {
        const GLUvec4 vn =
          get_negative_far_point(planes[i], aabb);
        if (gluDot3(vn, planes[i].n) + planes[i].d > 0)
            return OUTSIDE;

        const GLUvec4 vp =
          get_positive_far_point(planes[i], aabb);
        if (gluDot3(vp, planes[i].n) + planes[i].d > 0)
            intersect = true;
    }

    return (intersect) ? INTERSECTING : INSIDE;
}
```

AABB / Frustum Intersection

- References:
  - http://www.ce.chalmers.se/~uffe/vfc_bbox.pdf
  - http://www.ce.chalmers.se/~uffe/vfc.pdf

OBB / Frustum Intersection

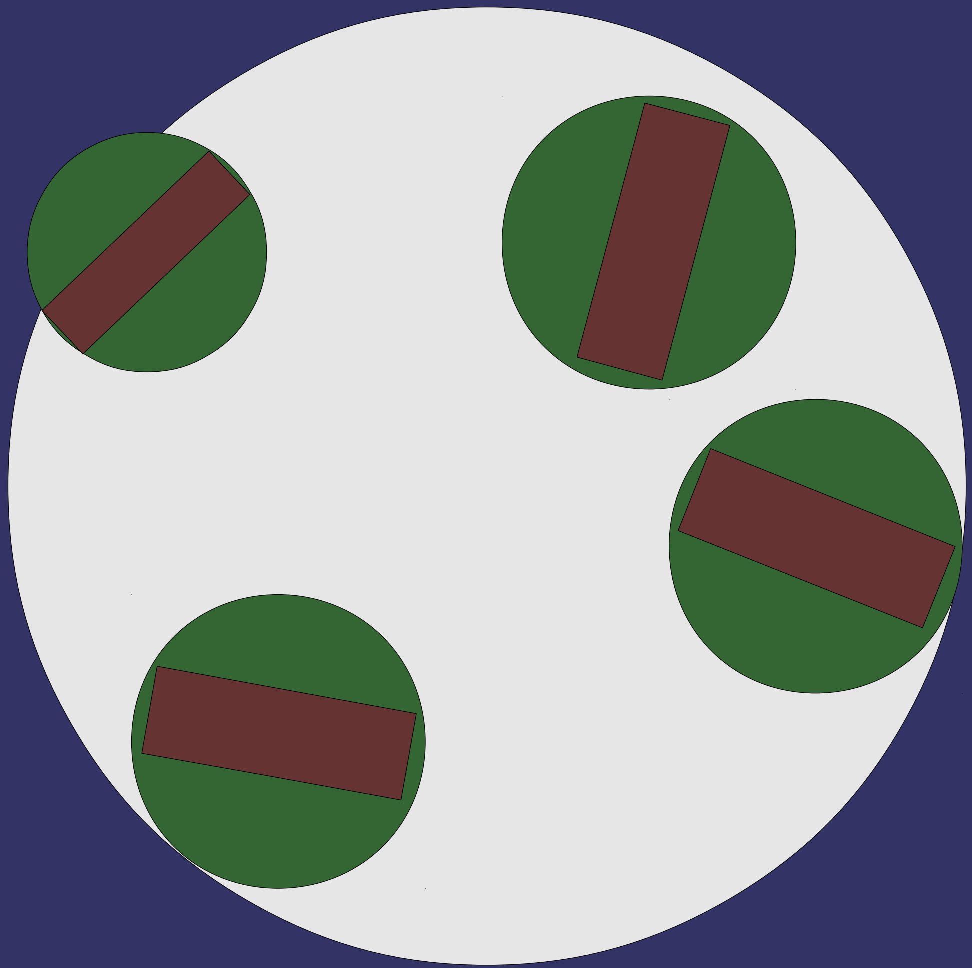
- Same!
  - Transform the frustum to the coordinate space of the OBB
  - Effectively makes the OBB to an AABB

BV Hierarchies

- Bounding volume containing bounding volumes containing bounding volumes, etc.
  - Arrange the BVs in a tree-like structure
  - Sibling BVs may occupy overlapping space

BV Hierarchies

- Parent-child property:
  - Each parent BV contains its child BVs
  - Not required, but makes somethings easier
    - Parent BV need only contain objects in child BVs
    - Top level circle (right) contains all boxes but not all sub-circles.



Desirable BVH Characteristics

- Nodes within a subtree should be “near” each other
  - Farther down the tree, the nodes should be closer

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Desirable BVH Characteristics

- Nodes within a subtree should be “near” each other
  - Farther down the tree, the nodes should be closer
- Each node should be tight-fitting
  - Just like non-hierarchical bounding volumes
- Nodes near the root are more important than nodes near the leaves
  - Trivial reject (or trivial accept) as many objects as possible as with as little work as possible

Desirable BVH Characteristics

- Minimal overlap of sibling nodes
  - Overlap can force traversal of multiple subtrees

Desirable BVH Characteristics

- Minimal overlap of sibling nodes
  - Overlap can force traversal of multiple subtrees
- Hierarchy should be balance w.r.t. node structure and content
  - Balanced structure just like regular search trees
  - Balanced content (i.e., number of objects in nodes) allows earlier trivial rejection

Desirable BVH Characteristics

- Minimal overlap of sibling nodes
  - Overlap can force traversal of multiple subtrees
- Hierarchy should be balance w.r.t. node structure and content
  - Balanced structure just like regular search trees
  - Balanced content (i.e., number of objects in nodes) allows earlier trivial rejection
- Worst-case performance should not be much worse than average-case performance
  - Avoid stuttering framerates

Desirable BVH Characteristics

- Generate *without* human intervention
  - Automatically generate without artist or programmer guiding the process

Desirable BVH Characteristics

- Generate *without* human intervention
  - Automatically generate without artist or programmer guiding the process
- Memory overhead should be low
  - Just like non-hierarchical bounding volumes

BVH Creation

- Three common strategies:
  - Insertion
  - Top-down
  - Bottom-up

BVH Creation – Top-Down

- Start with single BV and recursively subdivide
  - Easy to implement
  - Doesn't result in optimal BVH

BVH Creation – Top-Down

```
BVHNode *build_BVH(Entity *e, int num_e)
{
    BoundingVolume *bv = new BoundingVolume(e, num_e);
    BVHNode *node = new BVHNode(bv);

    if (num_entity < threshold) {
        node->is_leaf = true;
    } else {
        int first_half_count = divide_entities(e, num_e);
        node->child[0] = build_BVH(& e[0],
            first_half_count);
        node->child[1] = build_BVH(& e[first_half_count],
            num_e - first_half_count);
    }

    return node;
}
```

BVH Creation – Top-Down

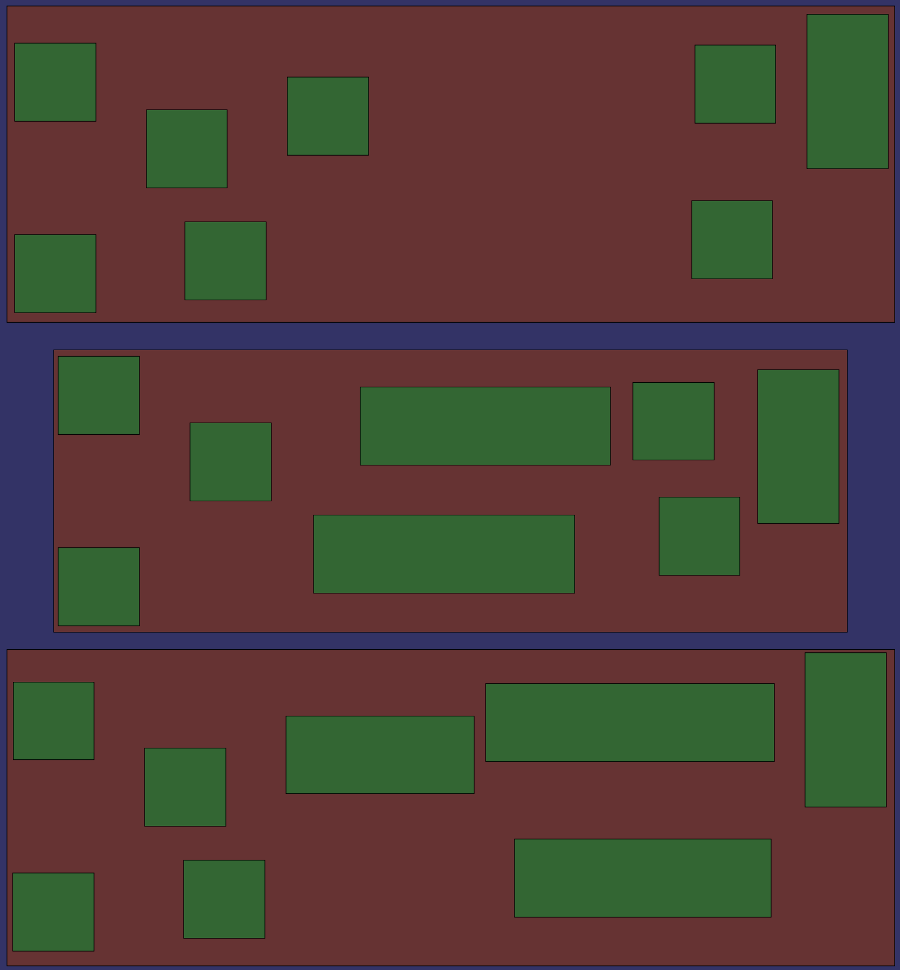
- The key element is `divide_entities`
  - As coded, assumes each entity is in exactly one set
  - *Not* the only strategy
- How do we decide where to divide the set?

BVH Creation – Top-Down

- Median-cut is a common strategy
  - Select an axis
    - Longest axis of the BV being partitioned is a common choice
  - Project all entities onto this axis
  - Sort projected entities by position
  - First half goes in the first node, second half goes in the second node

BVH Creation – Top-Down

- Median-cut is easy to implement, but it poorly partitions some sets:



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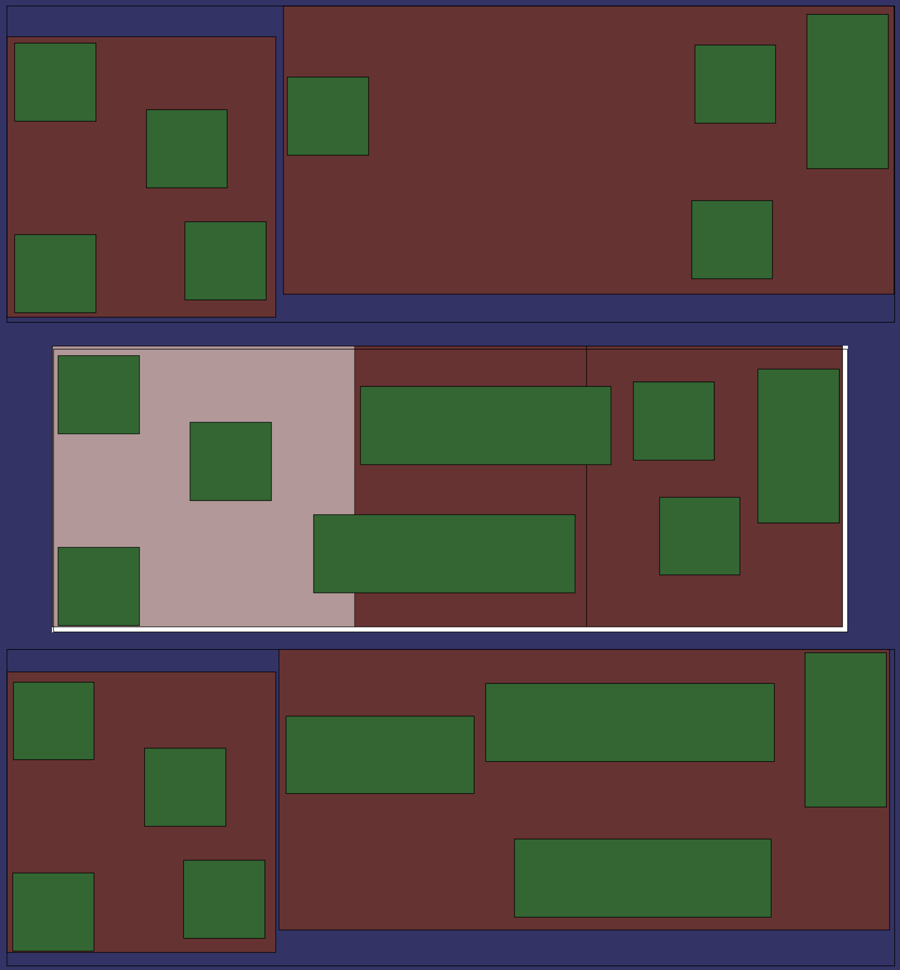

Too much empty space

Too much overlap

Unbalanced node sizes

BVH Creation – Top-Down

- Other heuristics:
  - Minimize sum of volumes
  - Minimize largest volume
  - Minimize overlap volume
  - Maximize child node separation
- No single heuristic is perfect
  - Implement a primary heuristic and adjust choice if secondary heuristic scores very poorly
  - Repeat for all heuristics or until a heuristic passes without adjustment

BVH Creation – Top-Down

- Infinite number of possible partition axes
  - Similar to the problem of selecting the basis of OBB
- Common choices:
  - Aligned axes of BV
  - Axes of parent BV
  - Axis through most distant points
  - Axis of greatest variance

BVH Creation – Top-Down

- Once an axis is selected, a split-point must also be selected
  - Median of projected object centroids
  - Mean of projected object centroids
  - Median of projected BV extents
  - Pick best of n evenly spaced points along axis

BVH Creation – Bottom-Up

- Repeatedly merge individual BVs:
  - Create a BV for each object
    - Store in an “active” BV list
  - Select 2 or more BVs to merge
    - Remove old BVs from active list
    - Add new, merged BV to active list
  - Lather, rinse, repeat until only one BV remains
- Tradeoffs:
  - Often much, much slower
  - More complex implement
  - Usually results in *much* better hierarchies

BVH Creation – Bottom-Up

- The key element is the algorithm for node selection

BVH Creation – Bottom-Up

- The key element is the algorithm for node selection
- Obvious, brute-force approach: search active list for pair of nodes that form least-volume BV
  - $O(n^2)$ for the search repeated $(n-1)$ times: $O(n^3)$ for the lose. :(

BVH Creation – Bottom-Up

- The key element is the algorithm for node selection
- Obvious, brute-force approach: search active list for pair of nodes that form least-volume BV
  - $O(n^2)$ for the search repeated $(n\text{-}1)$ times: $O(n^3)$ for the lose. :(

Other heuristics can also be used

BVH Creation – Bottom-Up

- Use the brute-force method as basis for an improved method:
  - For each node, determine the best node for it to pair with
    - Store both nodes with heuristic score in a priority queue
  - Loop, removing the head from the queue:
    - Validate stored size
      - May have changed if either node was already removed
    - If size is still smallest, calculate pairing for new node and add to queue
    - Otherwise, re-insert the original node in the queue

BVH Creation – Insertion

- Find location to insert node with least cost
  - Heuristic is usually along the lines of volume added to BV and all parent BVs
  - Large objects will be added near the root, small objects will be added near the leaves
  - Far away (isolated) objects will be added near the root

BVH Creation – Insertion

- Common insertion strategies:
  - Depth first:
    - At each step, pick the child with the least cost.
    - Recur on its children
    - Search cost is $O(\ln n)$ with n searches → $O(n \ln n)$
  - Guided breadth first:
    - Keep track of cost at each visited depth, recur on branch with current best cost
    - Worst-case search cost is $O(n) \rightarrow O(n^2)$
      - Average case is still $O(n \ln n)$
    - Results in much better tree
      - Uses global information instead of just local information

Next week...

- Quiz #3
- Texture mapping, part 1

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