VGP351 – Week 3

- Agenda:
  - Quiz #1
  - Transformations
    - Modeling
    - Viewing
    - Projection

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Coordinate Spaces

- Is the spaceship moving, or is the viewer moving?

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Coordinate Spaces

- Relativistically, it *doesn't matter*
  - Pick the reference frame that's most convenient at the time

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Coordinate Spaces

- Coordinates are always relative to some “space”
  - Object space: Local coordinate system of the object
  - World space: Global coordinate system relative to the 3D “world”
  - Eye / camera space: Coordinate system relative to the viewer
- When we translate objects relative to other objects, we may talk about other spaces
  - If the hand of a 3D model is rotated relative to the arm of the model, we may talk about “hand-space” or “arm-space”

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Coordinate Spaces

- Watch your coordinate spaces!
  - When performing calculations, be sure that the coordinate spaces match
    - Measuring distances between points
    - Measuring angles between vectors
    - Performing transformations
  - Just like being careful of units in physics / chemistry equations
    - If an acceleration calculation comes out in Newtons (kg m/s^2) instead of m/s^2, you *know* there's an error

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Coordinate Spaces

- Variable names should convey the coordinate space

```
vec3 normal_ws;     // normal in world-space
vec4 position_es;   // position in eye-space
vec4 light_ws;      // light pos in world-space
vec3 light_dir_ss;  // light direction in surface-
                    // space

// Obviously wrong!
light_dir_ss = (light_ws – position_es).xyz;

// Not obvious, but still wrong
light_dir = (light – position).xyz;
```


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Orthonormal Basis

- It's a mouthful...what does it mean?
- A vector space where all of the components are *orthogonal* to each other, and each is *normal*
  - Normal meaning unit length
  - Orthogonal meaning at right angles
    - The *other* meaning of normal
- Every pure rotation matrix (i.e., no scaling) is an orthonormal basis
  - As is the identity matrix

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Viewing

- Q: Given a world position for a camera, a world position to point the camera at, and an “up” direction, how can we construct a transformation using just rotations and translations?

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Viewing

- Q: Given a world position for a camera, a world position to point the camera at, and an “up” direction, how can we construct a transformation using just rotations and translations?
- A: We can't. We need 3 vectors to construct an orthonormal basis
  - [Hughes 99] presents a method to construct from just one vector, but it has limitations

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Viewing

- Given:
  - $\mathbf{e}$: Position of the eye (or camera) in world-space
  - $\mathbf{v}$: The point being viewed
  - $\mathbf{u}$: the “up” direction
- Calculate the unit vector from the viewpoint to the eye:

$$\mathbf{f}' = \mathbf{v} - \mathbf{e}$$

$$\mathbf{f} = \frac{\mathbf{f}'}{|\mathbf{f}'|}$$

  - This is the Z axis

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Viewing

- Calculate a vector orthogonal to the Z-axis and the up vector:

$$\mathbf{s} = \mathbf{f} \times \mathbf{u}$$

  - This is the X-axis

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Viewing

- Calculate a vector orthogonal to the Z-axis and the up vector:

$$\mathbf{s} = \mathbf{f} \times \mathbf{u}$$

  - This is the X-axis
- Calculate a vector orthogonal to the X-axis and the Z-axis:

$$\mathbf{t} = \mathbf{s} \times \mathbf{f}$$

  - This is the Y-axis
  - Why can't we just use $\mathbf{u}$?

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Viewing

- Drop these vectors into a matrix:

$$\mathbf{M}_\mathrm{v} = \begin{bmatrix} \mathbf{s}_0 & \mathbf{s}_1 & \mathbf{s}_2 & 0 \\ \mathbf{t}_0 & \mathbf{t}_1 & \mathbf{t}_2 & 0 \\ -\mathbf{f}_0 & -\mathbf{f}_1 & -\mathbf{f}_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & -\mathbf{e}_0 \\ 0 & 1 & 0 & -\mathbf{e}_1 \\ 0 & 0 & 1 & -\mathbf{e}_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - The translation moves the eye to the origin

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References

General information about rotation matrices and orthonormal bases:

http://en.wikipedia.org/wiki/Rotation_matrix

http://www.wikipedia.org/Orthonormal_basis

Really good explanation of arbitrary rotation matrices:

http://www.euclideanspace.com/maths/geometry/rotations/conversions/angleToMatrix/index.htm

Hughes, J. F., and Möller, T. Building an Orthonormal Basis from a Unit Vector. *Journal of Graphics Tools* 4, 4 (1999), 33-35. http://www.cs.brown.edu/research/pubs/authors/john_f._hughes.html

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Projection

- Once objects are transformed to camera-space, they're still 3D
  - The screen is still 2D
  - Camera parameters (e.g., field of view) need to be applied
- Four steps remain:
  - Projection from camera space to clip coordinates
    - A cube on the range ±1
  - Perspective divide
  - Map clip coords to normalized device coords (NDC)
    - X and Y in ±1, Z in [0,1]
  - Map NDC to pixel coordinates

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Projection

- Perspective:
  - Simulates visual foreshortening caused by the way light projects onto the back of the eye
  - Represents the view volume with a frustum (a pyramid with the top cut off)
  - The real work is done by dividing X and Y by Z
- Orthographic:
  - Represents the view volume with a cube
  - Also called *parallel projection* because lines that are parallel before the projection remain parallel after

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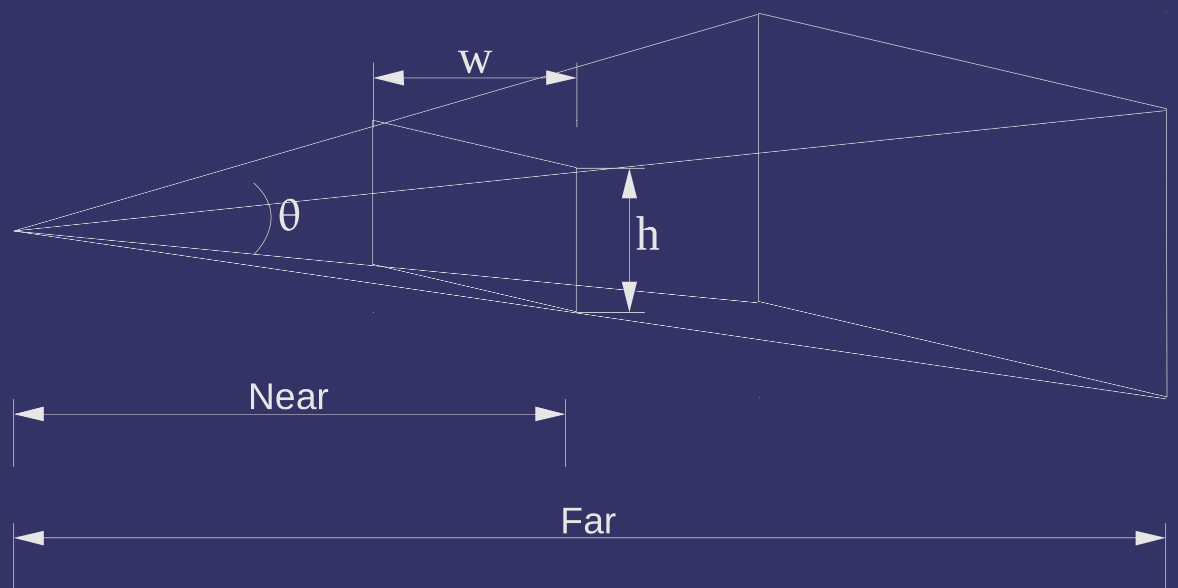
Perspective Projection

- A few parameters control the view volume:
  - Near: Distance from the camera to the near viewing plane. Objects in front of this plane will be clipped
  - Far: Distance from the camera to the far viewing plane. Objects behind this plane will be clipped
  - θ: Field-of-view in the Y direction
  - Aspect ratio: Ratio of the width of the view to the height of the view



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Perspective Projection

$$f = cotan\left(\frac{\theta}{2}\right)$$

$$\mathbf{M}_p = \begin{bmatrix} \frac{f}{aspect} & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & -\frac{2\times far\times near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

- Limited form of projection matrix that assumes symmetry in X and Y directions
- *near* and *far* are distances
  - We're actually looking down the negative Z axis in camera space

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Perspective Projection

- **WARNING:**
  - `near` and `far` are reserved words in MS compilers
    - Nice of them to follow the rules of the C specification
    - Dates back to quirks of the old 8086 and 80286 CPUs
  - Maybe use:
    - `hither` and `yon`
    - `zNear` and `zFar`

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Putting it all together

- Typically have a *modeling* transform, a *viewing* transform, and a *projection*
  - Combine these into a single "model-view-projection" matrix: $\mathbf{M}_{mvp} = \mathbf{M}_p \times \mathbf{M}_v \times \mathbf{M}_m$
  - Transform a vertex by this single matrix:

```
uniform mat4 mvp;
void main(void)
{
        gl_Position = mvp * gl_Vertex;
}
```


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References

http://en.wikipedia.org/wiki/3D_projection

- Especially the third step: perspective transform

http://en.wikipedia.org/wiki/Orthographic_projection_%28geometry%29

http://en.wikipedia.org/wiki/Isometric_projection

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Next week...

- Hidden surface removal / occlusion
  - Backface culling
  - Painters algorithm
  - Z-buffer
  - Frustum culling
- Assignment #2, part 1

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