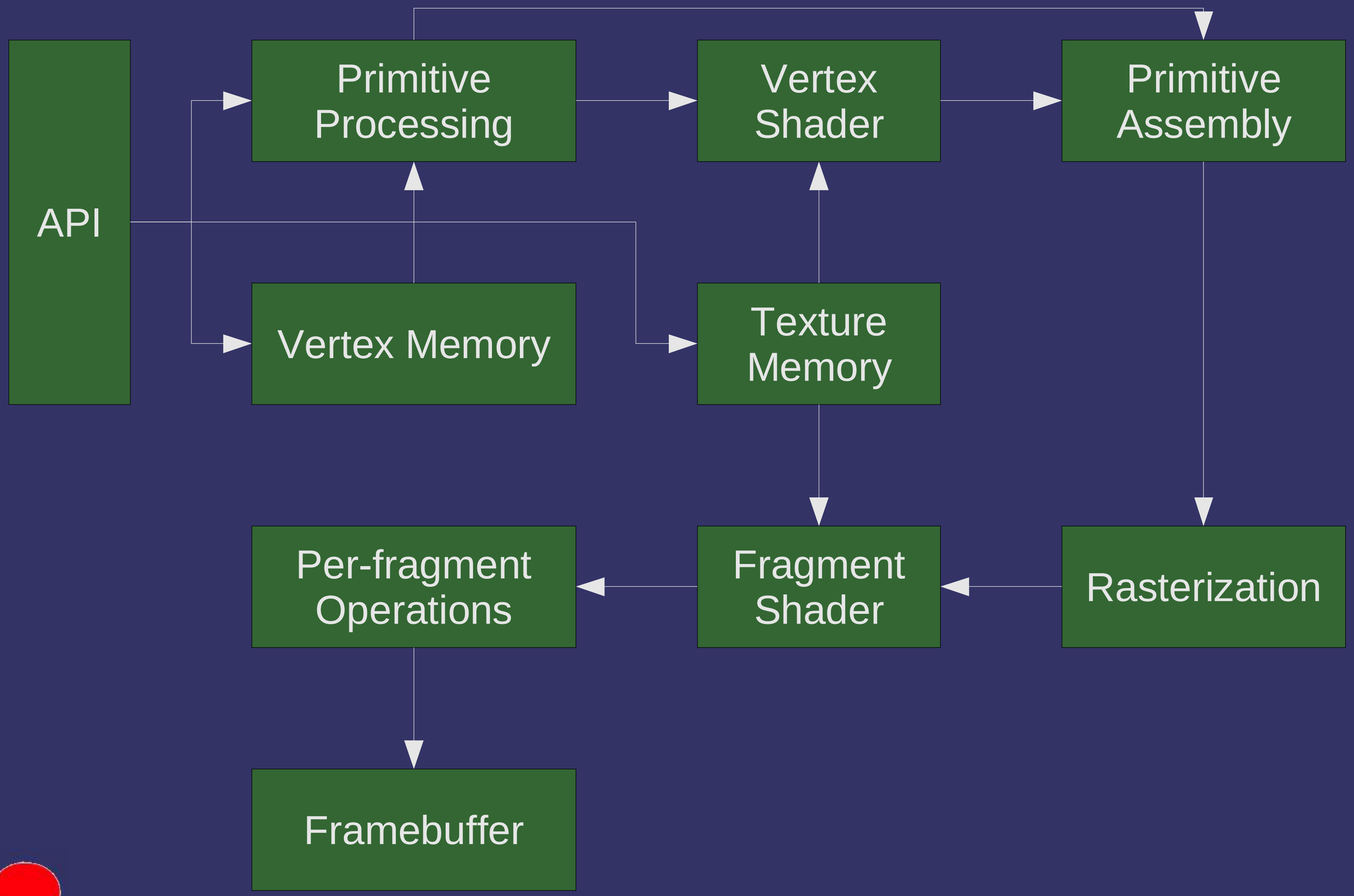
# *VGP351 – Week 2*

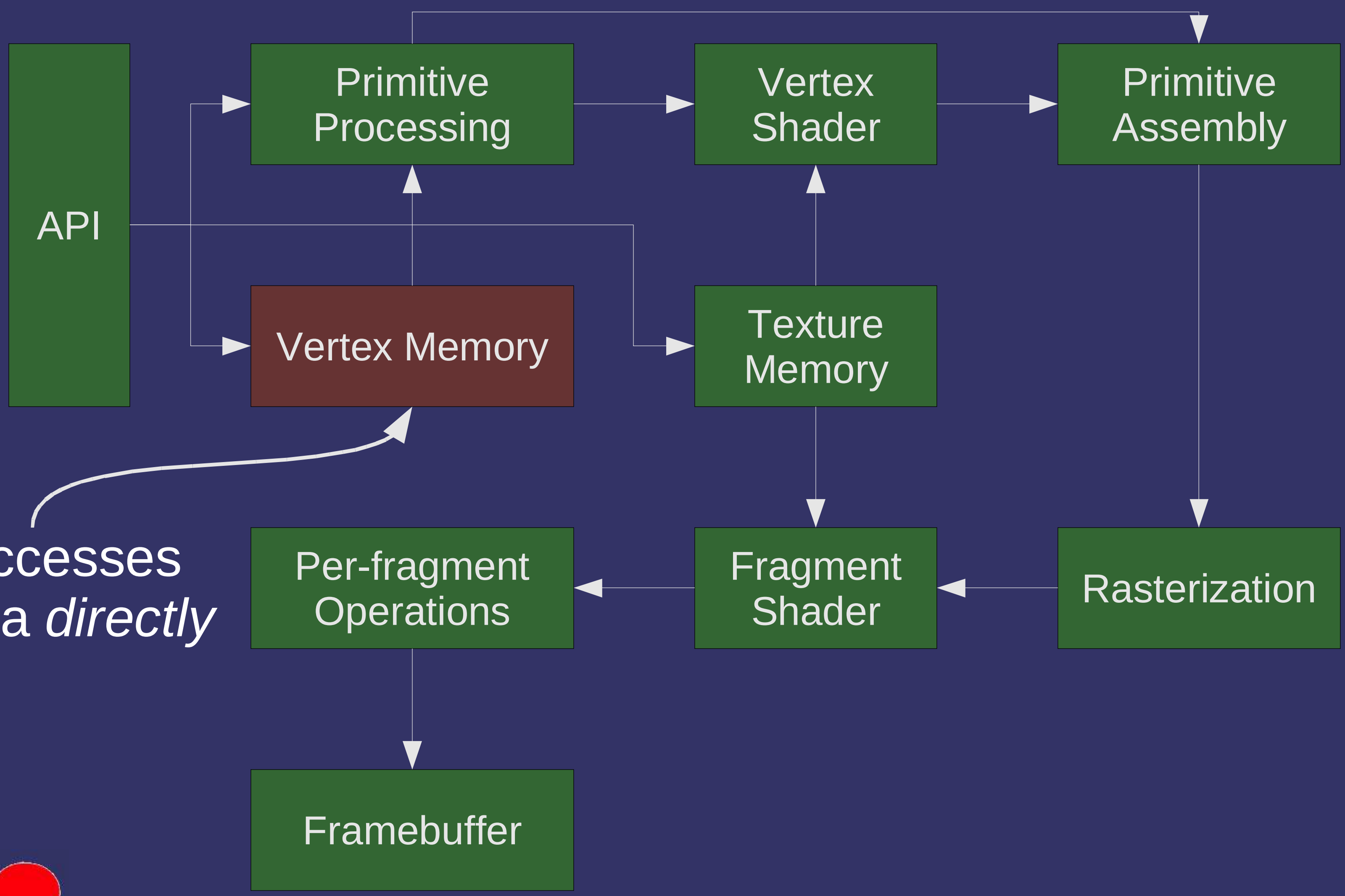
- Agenda:
  - Getting data to the GPU
    - Buffer objects
    - Vertex attributes
    - Uniforms
  - Types of primitives
  - Transformations
    - Modeling
    - Viewing
    - Projection

12-October-2011

# Graphics Pipeline



# *Graphics Pipeline*




GPU accesses
this data *directly*

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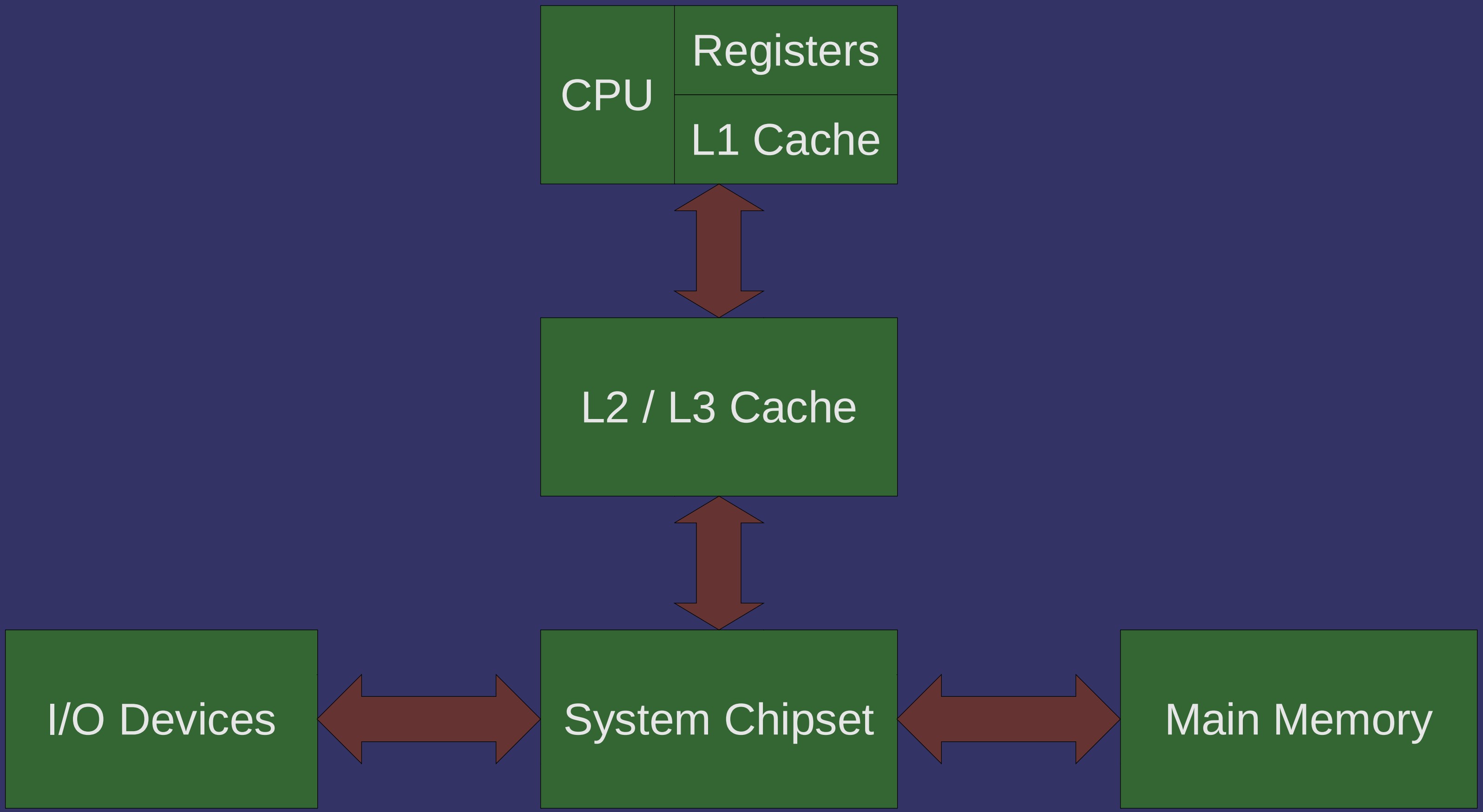
# *Memory Architecture*

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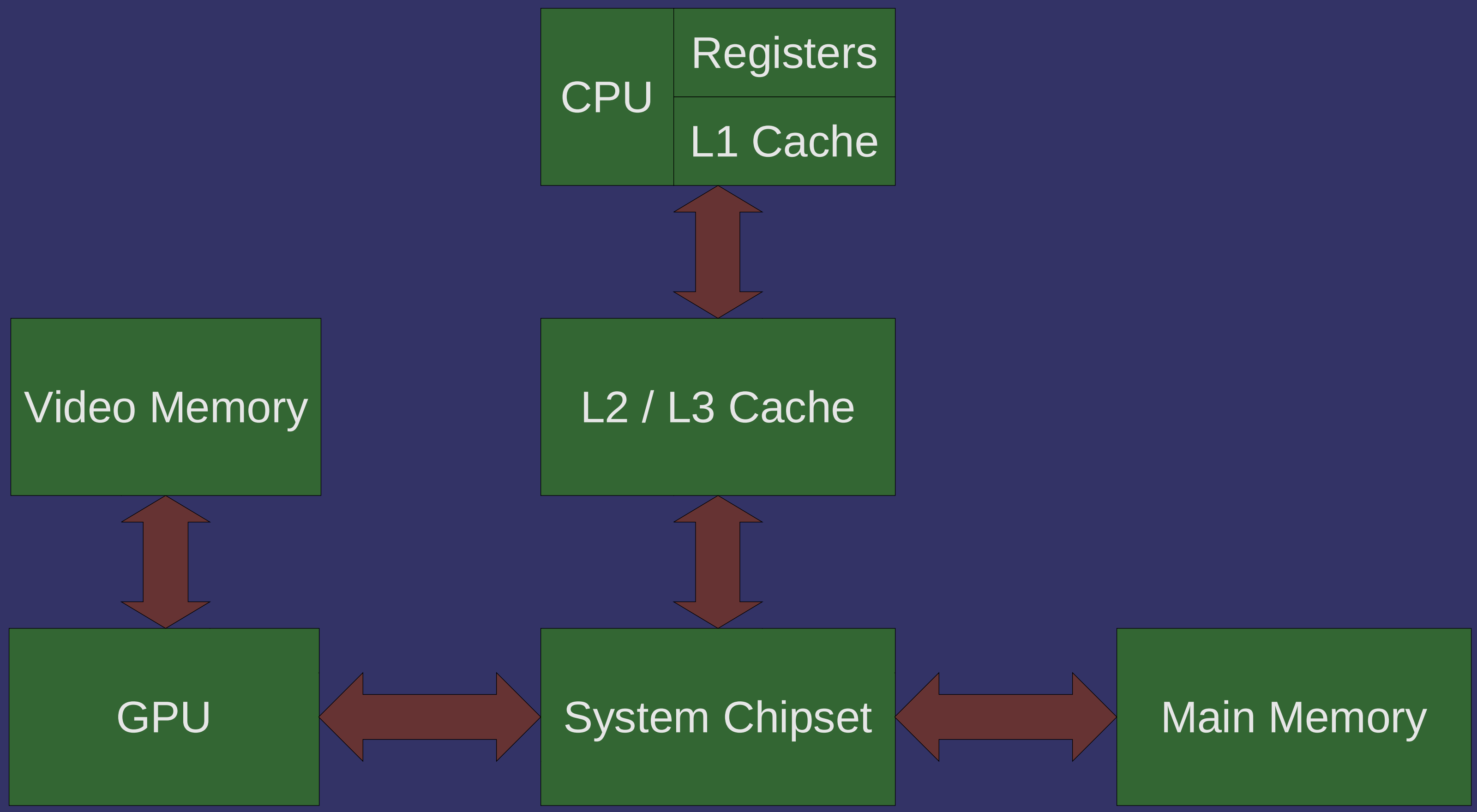
# *Memory Architecture*

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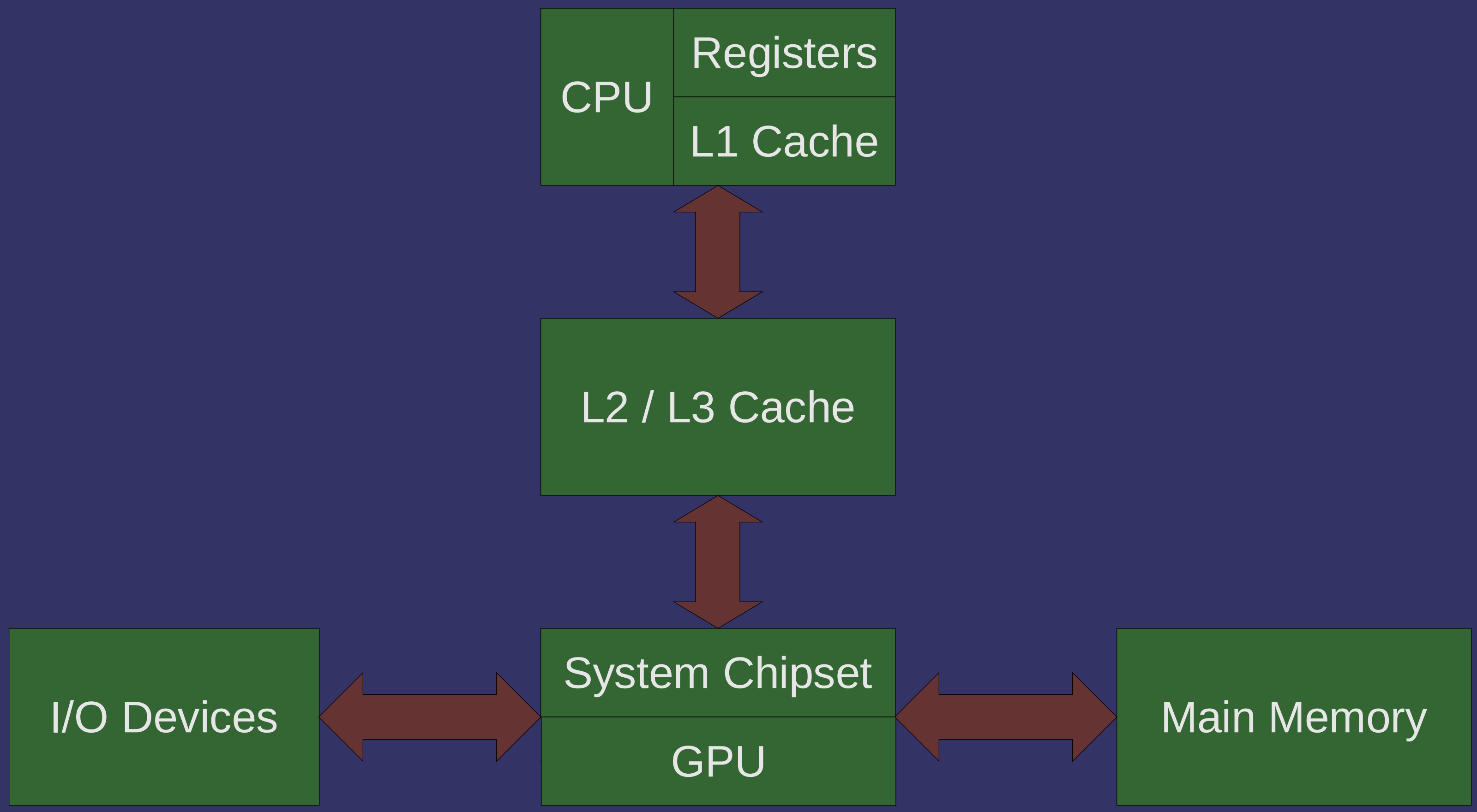
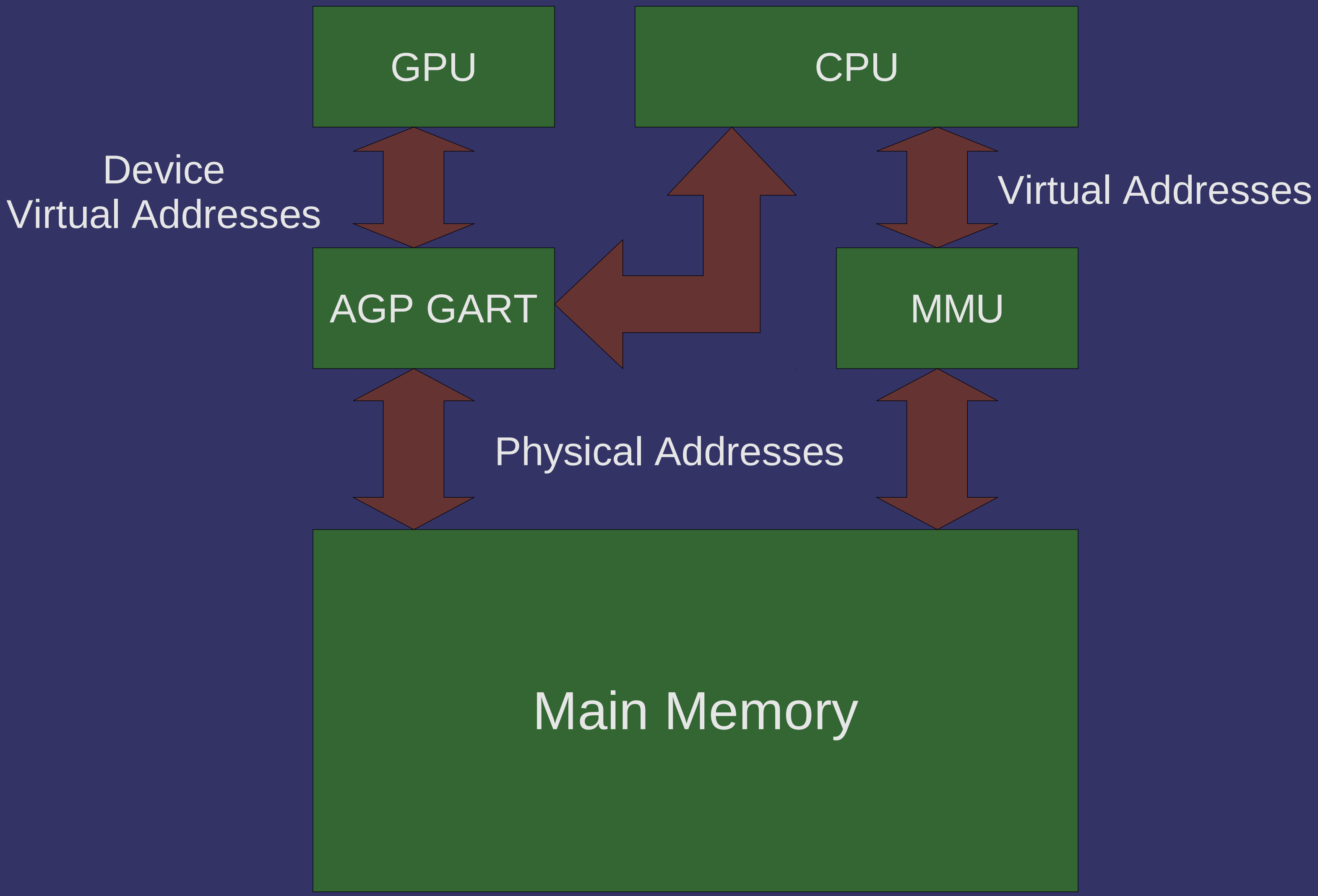
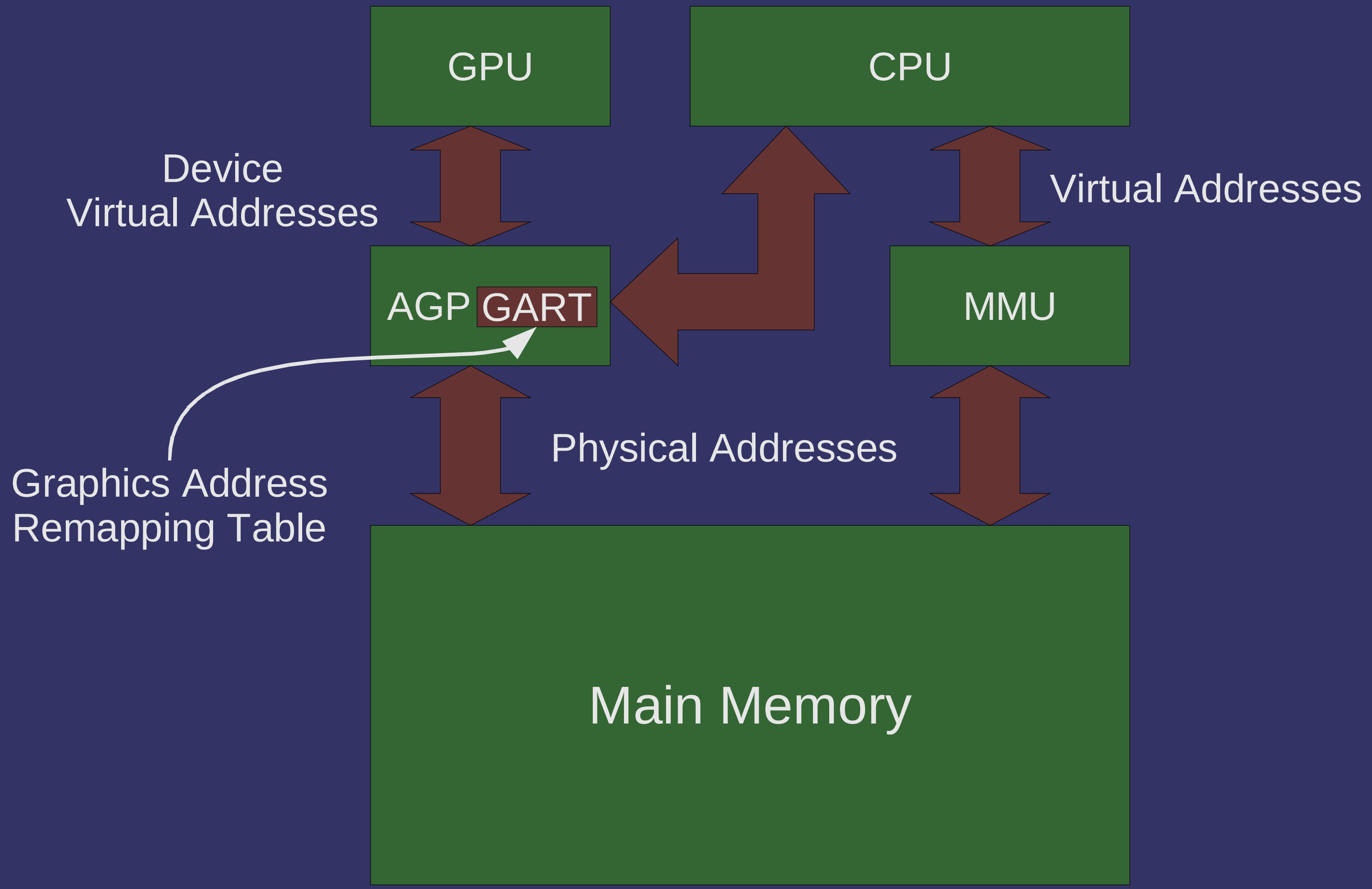
# Unified Memory Architecture



# Memory Map


GPU
CPU
Device
Virtual Addresses
Virtual Addresses
AGP GART
MMU
Physical Addresses
Main Memory

# Memory Map

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# *Vertex Memory*

- Practically, the GPU can only access:
  - Memory physically on the graphics card
  - Memory mapped in the GART
- Only the driver can provide GART or card memory
  - The driver controls what's available to the GPU
  - The driver knows how much memory is available
  - The driver controls the GART mappings
  - The driver knows which memory pool to use
    - ...but we have to give it some hints

# *Vertex Memory*

- In OpenGL this memory is called *buffer object*
  - Used somewhat like a file:
    - Accessed via an opaque handle
    - Bulk I/O “accessor” routines
    - Direct mapping and access via a pointer
      - See http://en.wikipedia.org/wiki/Memory-mapped_file

# Buffer Objects

- Generate “names” for the buffer objects:

  `glGenBuffers(GLsizei num, GLuint *names);`

- “Bind” a buffer for use:

  `glBindBuffer(GLenum target, GLuint name);`

  – `target` selects which buffer we're talking about
    – `GL_ARRAY_BUFFER` is used for vertex data
    – `GL_ELEMENT_ARRAY_BUFFER` is used for vertex indices
      – More on that and other targets later in the term
  – Binding creates the object, but it still has no storage
    – Like creating an empty file

# *Buffer Objects*

- Storage is created and *optionally* initialized with:

```
void glBufferData(GLenum target,
    GLsizeiptr size, const GLvoid *data,
    GLenum usage);
```

  – `usage` tells the GL how the app will utilize the buffer

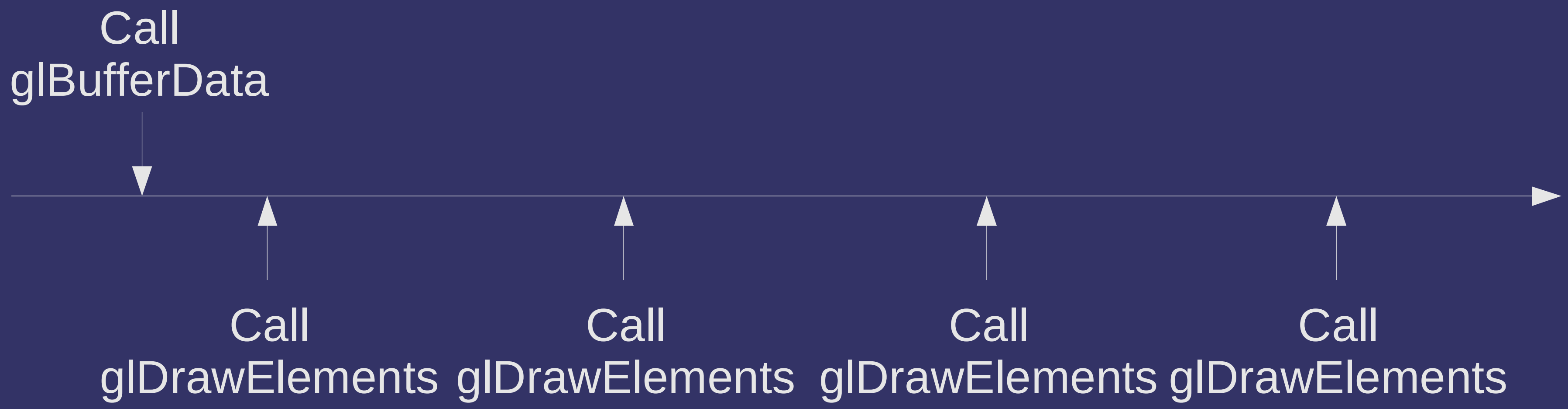
- Storage is updated with:

```
void glBufferSubData(GLenum target,
    GLintptr offset, GLsizeiptr size,
    const GLvoid *data);
```

# *Buffer Object Usage Hints*

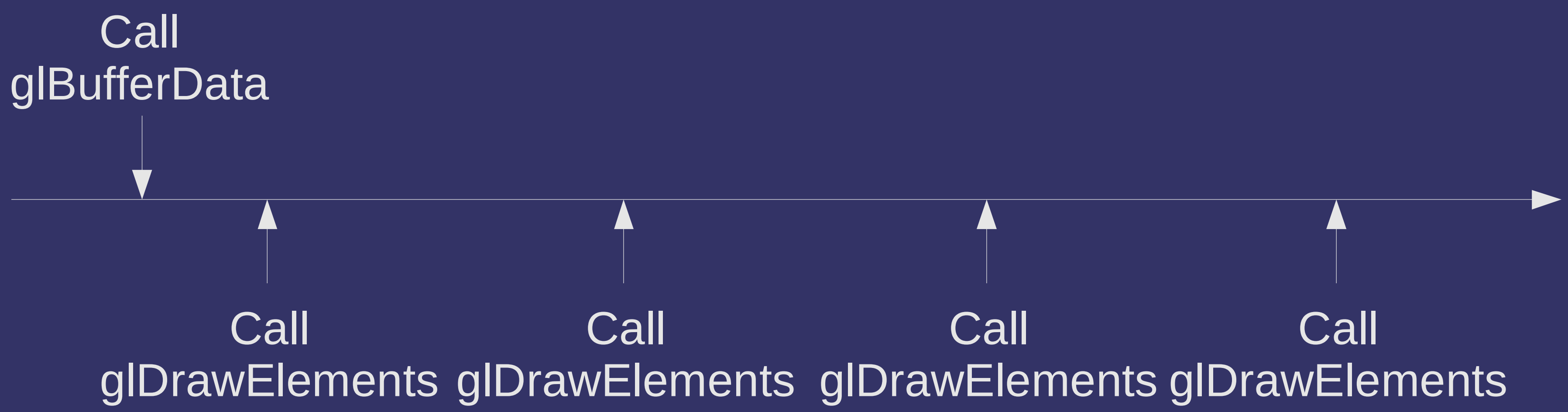
- Usage conveys information along two axes:
  - Data “frequency”:
    - Stream – data is specified once and used a few times
    - Static – data is specified once and used many times
    - Dynamic – data is specified and used many times
  - Data “usage”:
    - Draw – data used as source for drawing
    - Read – data copied from GL and read back to client
    - Copy – data copied from GL and used as source for drawing
  - Combine these to create the enums (e.g., `GL_STATIC_DRAW`)

# *Buffer Object Usage Hints*

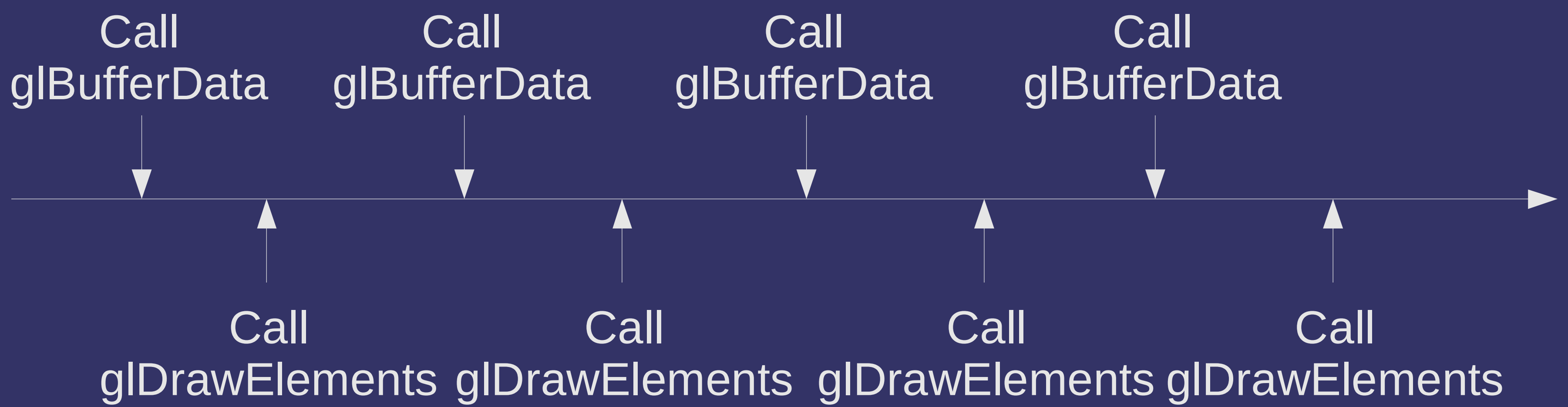


# *Buffer Object Usage Hints*

`GL_STATIC_DRAW`

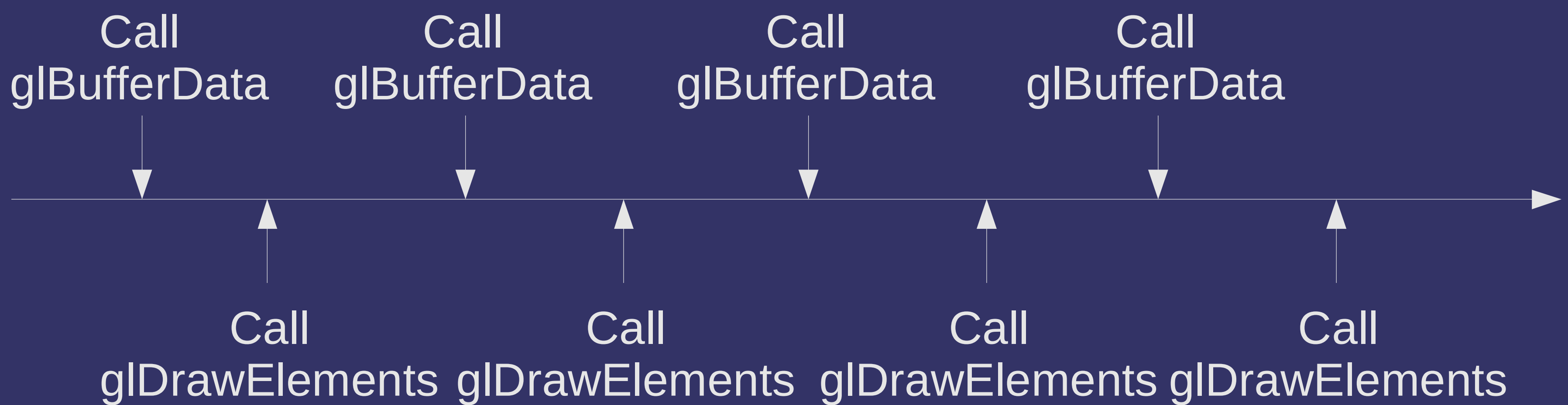


# Buffer Object Usage Hints



# *Buffer Object Usage Hints*

## `GL_STREAM_DRAW`

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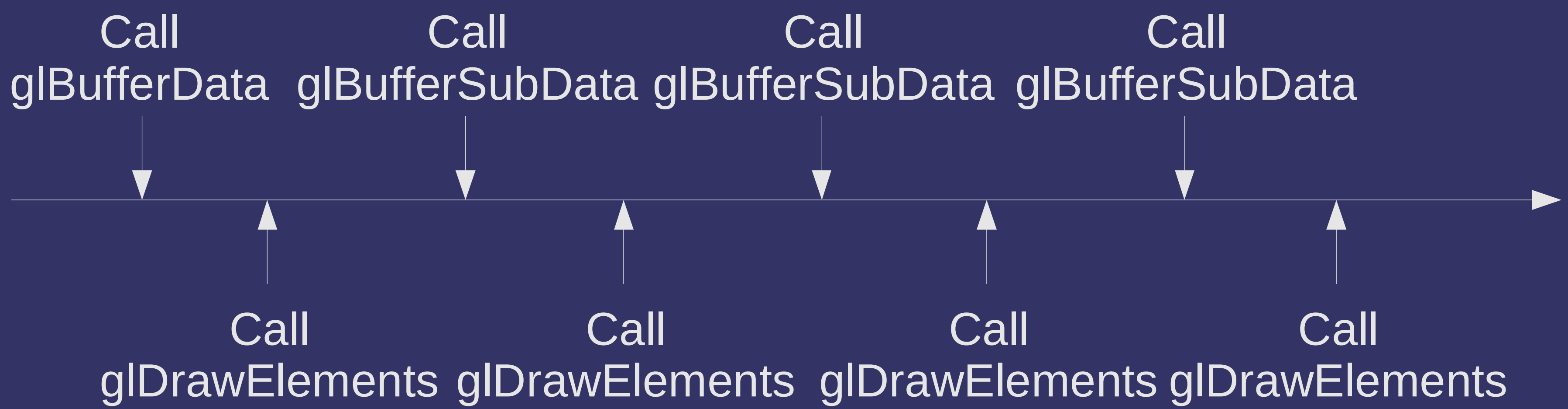
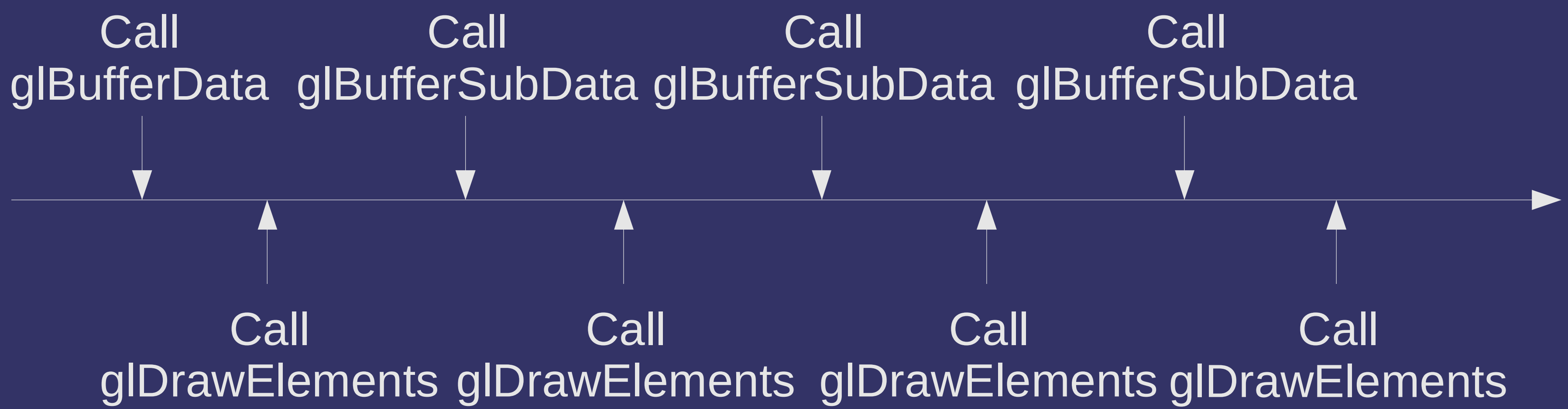
# Buffer Object Usage Hints



# *Buffer Object Usage Hints*

`GL_DYNAMIC_DRAW`



# Buffer Objects

- Memory backing the buffer can be mapped into CPU space:

```
GLvoid *glMapBuffer(GLenum target,
                    GLenum access);
```

  - `access` tells the driver how the application will access the mapped buffer:
    - `GL_READ_ONLY`
    - `GL_WRITE_ONLY`
    - `GL_READ_WRITE`
- Unmap the buffer with:

```
GLboolean glUnmapBuffer(GLenum target);
```

# *Now what?*

- The vertex data is in a buffer object...how do we tell the GPU know where to get it?

# *Vertex Attribute Pointer*

- Set the location and format of a vertex attribute with:

```
void glVertexAttribPointer(GLuint index,
    GLint size, GLenum type,
    GLboolean normalized, GLsizei stride,
    const GLvoid *pointer);
```

# *Vertex Attribute Pointer*

- Set the location and format of a vertex attribute with:

```
void glVertexAttribPointer(GLuint index,
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     const GLvoid *pointer);
```

In the API, attributes are numbered

# *Vertex Attribute Pointer*

- Set the location and format of a vertex attribute with:

```
void glVertexAttribPointer(GLuint index,
     GLint size, GLenum type,
     GLboolean normalized, GLsizei stride,
     const GLvoid *pointer);
```

Number of components in each element

Type of data (e.g., `GL_FLOAT`)

# *Vertex Attribute Pointer*

- Set the location and format of a vertex attribute with:

```
void glVertexAttribPointer(GLuint index,
    GLint size, GLenum type,
    GLboolean normalized, GLsizei stride,
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```

For integer data, specifies whether it is normalized or not

# *Vertex Attribute Pointer*

- Set the location and format of a vertex attribute with:

```
void glVertexAttribPointer(GLuint index,
    GLint size, GLenum type,
    GLboolean normalized, GLsizei stride,
    const GLvoid *pointer);
```

Number of bytes from
the start of one element
to the start of the next

# *Vertex Attribute Pointer*

- Set the location and format of a vertex attribute with:

```
void glVertexAttribPointer(GLuint index,
    GLint size, GLenum type,
    GLboolean normalized, GLsizei stride,
    const GLvoid *pointer);
```

Offset, in bytes, from the start of the buffer where the data starts

# *Enable Attribute*

- Attributes that will be used must also be enabled:

```
void glEnableVertexAttribArray(GLuint index);
```

- Attributes can later be disabled:

```
void glDisableVertexAttribArray(GLuint index);
```

# *Setting Attribute Numbers*

- GLSL uses names for attributes:

```
attribute vec4 color;
```

- The API uses numbers:

```
void glVertexAttribPointer(GLuint index,
    GLint size, GLenum type,
    GLboolean normalized, GLsizei stride,
    const GLvoid *pointer);
```

- How do we connect the two?

# Setting Attribute Numbers

- Bind the attribute name to the index we want:

```
void glBindAttribLocation(GLuint programObj,
    GLuint index, const GLchar *name);
```

  - Can only call *before* linking the program
  - Changes to attribute locations do not take effect until the program is linked (or linked again)

# *Drawing*

- Draw a series of vertices:

```
void glDrawArrays(GLenum mode, GLint first,
    GLsizei count);
```

# *Drawing*

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void glDrawArrays(GLenum mode, GLint first,
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```

Sets the primitive type

# Drawing

- Draw a series of vertices:

```
void glDrawArrays(GLenum mode, GLint first,
    GLsizei count);
```

Number of vertices to draw

Selects which vertex in the buffer to start drawing with

# *Primitive Types*

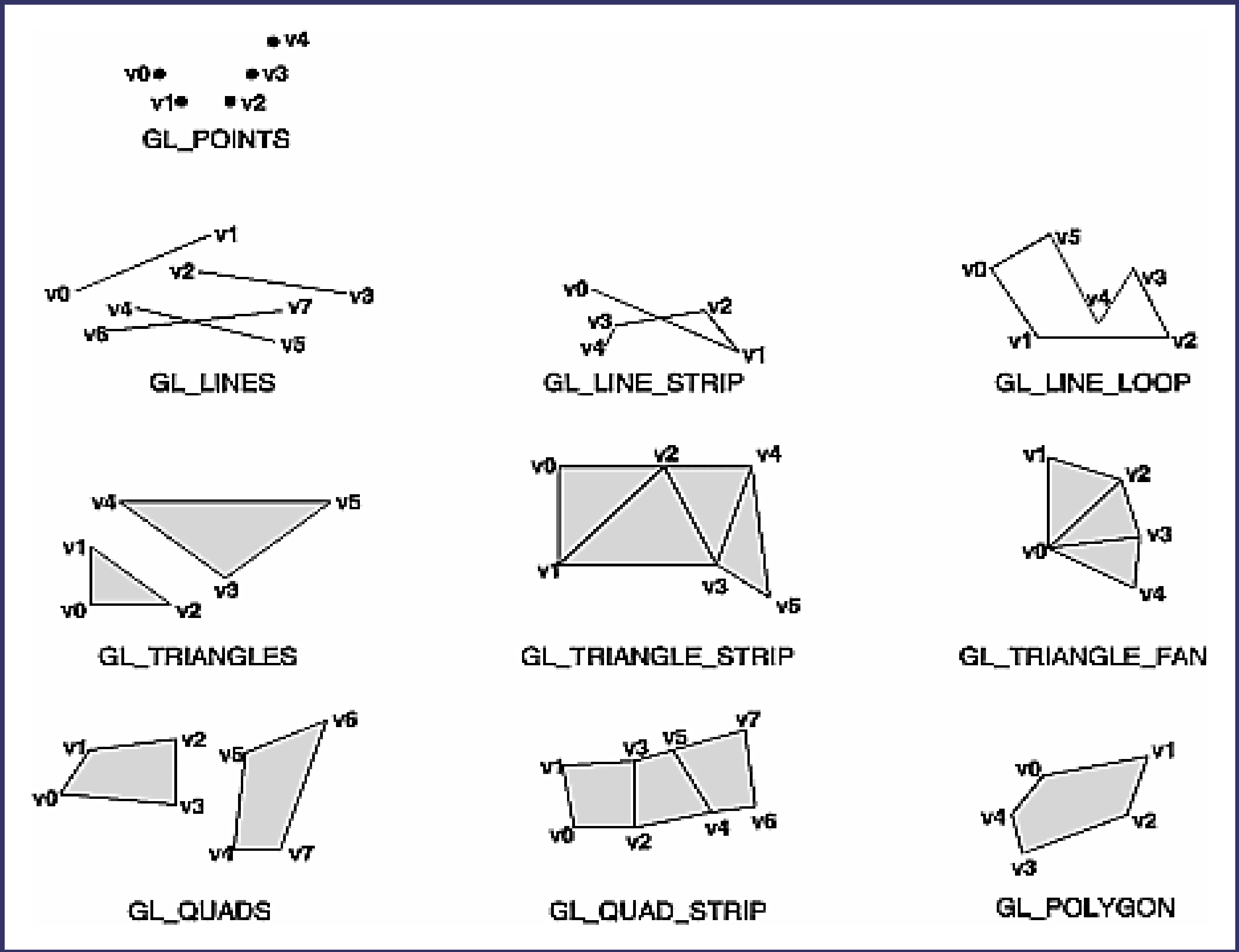


Image borrowed from "OpenGL Programming Guide".

# *Primitive Types*

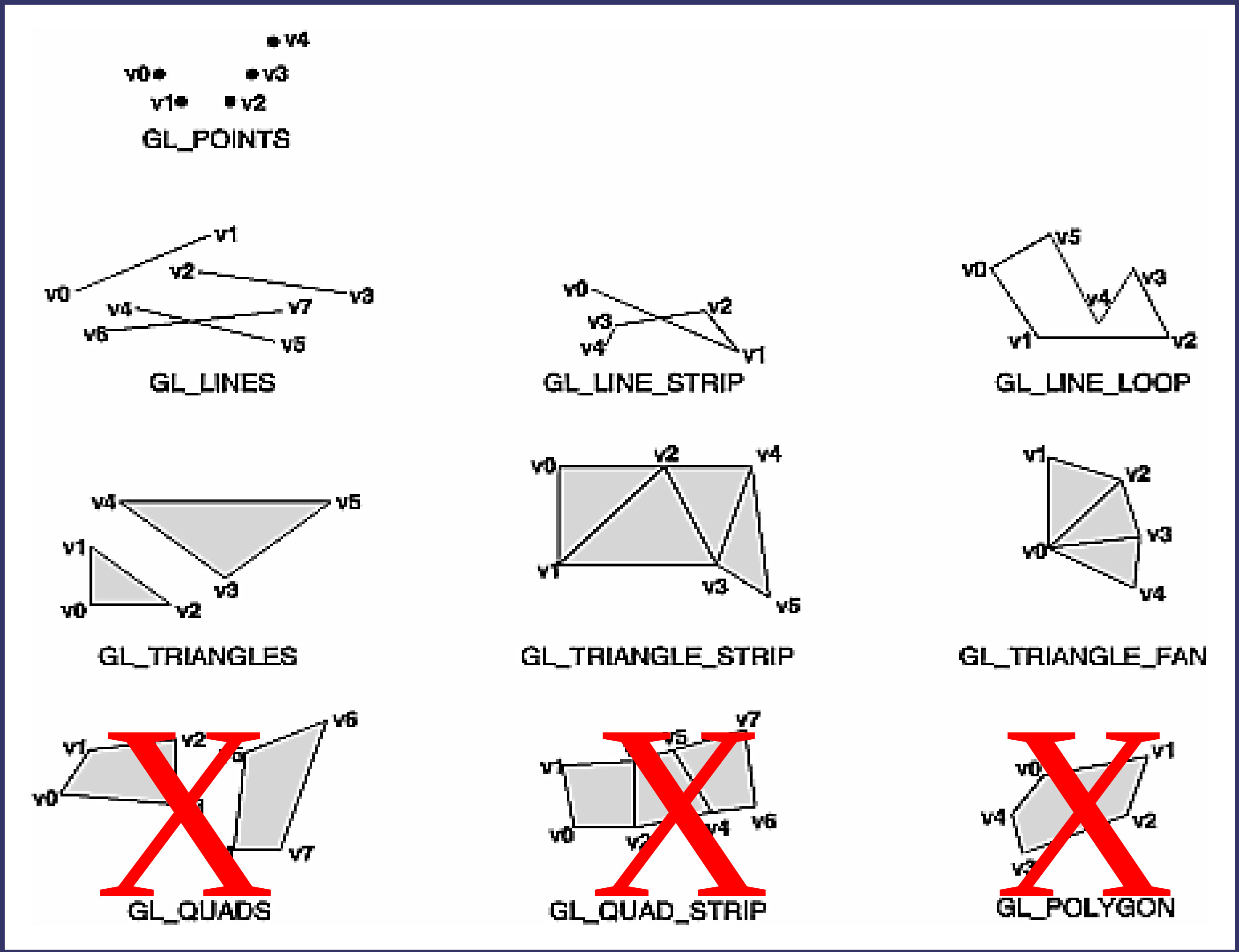


Image borrowed from “OpenGL Programming Guide”.

# *Uniforms*

- Data that changes *at most* per draw call
  - Set using `glUniform` and `glUniformMatrix` commands

# Uniforms

- Single elements can be set:

```
void glUniform4f(GLint location, GLfloat v0,
    GLfloat v1, GLfloat v2, GLfloat v3);
```

  - Also `1f`, `2f`, `3f`, `1i`, `2i`, `3i`, and `4i` variants
  - Number of fields and base type must match the uniform's type
    - `4f` for `vec4`
    - `1i` for `int`
    - etc.

# *Uniforms*

- Multiple array elements can be set:

```
void glUniform4fv(GLint location,
    GLsizei count, const GLint *value);
```

  - Also `1fv`, `2fv`, `3fv`, `1iv`, `2iv`, `3iv`, and `4iv` variants
  - Number of fields and base type must match the uniform's type
  - `count` is the number of array elements to set

# *Uniforms*

- Matrices can be set:

```
void glUniformMatrix4fv(GLint location,
    GLsizei count, GLboolean transpose,
    const GLfloat *value)
```

  - Many variants for different matrix sizes
    - Size must match the dimension of the uniform
  - `count` is the number of matrices to set
  - `transpose` specifies whether the supplied data is column-major or row-major
    - The OpenGL default is **always** column-major

# *Uniform Locations*

- What is this “location”?

```
void glUniform4fv(GLint location,
    GLsizei count, const GLfloat *value)
```

  - Like the index for attributes, but set by the linker in-stead of the applicatoin
  - The linker assigns a slot for each *active* uniform

# *Uniform Locations*

- Query the location

```
GLint glGetUniformLocation(GLuint program,
    const GLchar *name);
```

  - `program` is a linked program object
  - `name` is the uniform to locate
    - Can be a scalar, vector, or matrix
    - Can be a whole array or an array element
    - Can be a field of a structure
    - Cannot be a whole structure.
  - Returns -1 if the requested uniform does not exist or is not active

# Uniform Locations

```
// vertex shader
uniform vec4 v;
uniform mat4 m;
attribute vec4 a;

void main() {
    gl_Position = m * a;
}

// fragment shader
uniform vec3 c;

void main() {
    gl_FragData[0] =
        vec4(c, 1.0);
}
```

# *Uniform Locations*

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These uniforms are active.

# *Uniform Locations*

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    gl_FragData[0] =
        vec4(c, 1.0);
}
```

This uniform is not.

These uniforms are active.

# References

- More information about I/O MMUs in general:

  http://en.wikipedia.org/wiki/IOMMU

- Nvidia whitepaper about using VBOs:

  http://developer.nvidia.com/object/using_VBOs.html

# *Linear Algebra Primer*

- Three important data types:
  - Scalar values
  - Row / column vectors
    - 1x4 and 4x1 are the most common sizes
  - Square matrices
    - 4x4 is the most common size...to match the 1x4 & 4x1 vectors

# Notation

- Try to use the same notation as the textbook:
  - Angle: $\theta$ (lower-case Greek)
  - Scalar: $s$ (lower-case, italic, serif)
  - Vector or point: $\mathbf{v}$ (lower-case, bold, serif)
    - Sometimes $\hat{\mathbf{u}}$ is used to differentiate vectors from points
  - Matrix: $\mathbf{M}$ (upper-case, bold, serif)
  - Plane: $\pi$: $\mathbf{n} \cdot x + d$ ($\pi$: a vector and a scalar)
  - Triangle: $\triangle\mathbf{abc}$ ($\triangle$ 3 points)
  - Line segment: $\mathbf{ab}$ (2 points)
  - Geometric entity: $A$ (upper-case, italic, serif)

# *Row Vectors*

- These are special matrices that have multiple columns but only one row
  - Example: $\begin{bmatrix} 5.0 & 3.14 & 37 \end{bmatrix}$
- Addition and subtraction is component-wise:
  - Example: $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 9 & 10 & 11 \end{bmatrix} = \begin{bmatrix} 10 & 12 & 14 \end{bmatrix}$
  - Both vectors must be the same size
- Operations with scalars also component-wise:
  - Example: $3.2 \times \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 3.2 & 6.4 & 9.6 \end{bmatrix}$
- Notice that vector multiplication is missing...

# Column Vectors

- These are special matrices that have multiple rows but only one column
  - Example: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
- Work just like row vectors
- Notationally convert a row to a column with a $\mathrm{T}$ in the exponent
  - Example: $\mathbf{v}^T$
  - We'll talk more about this notation later...

# *Vector Operations*

- There are a few operations specific to vectors that are really important to graphics:
  - Dot product
  - Vector magnitude / normalization
  - Cross product

# Dot Product

- Component-wise multiply, then sum components
  - Example:

$$\begin{bmatrix} 2.3 & 1.2 \end{bmatrix} \cdot \begin{bmatrix} 1.7 & 6.5 \end{bmatrix} = (2.3 * 1.7) + (1.2 * 6.5) = 11.71$$

  - Noted as $\mathbf{u} \cdot \mathbf{v}$ or $\langle \mathbf{u}, \mathbf{v} \rangle$
  - Also known as the *inner product* or *scalar product*

# *Vector Magnitude*

- Noted by vertical bars around the vector
  - Like absolute value...which is the scalar magnitude
  - Can also be thought of as the length of the vector
- Square-root of dot-product of vector with itself
  - Like absolute value
  - Example: $\left\| \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \right\| = \sqrt{\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}} =$

$$\sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{2}{4} + \frac{2}{4}} = 1$$

# *Normal*

- *Normal* is an overloaded term in graphics and linear algebra
  - Sometimes it means a vector has unit length
    - $|\mathbf{u}| = 1.0$
    - Can say the vector is “normalized”
  - Sometimes it means a vector is perpendicular to a surface or another vector
    - This mean the angle between the vectors is 90°
    - Can say that the vectors are “normal to each other”
    - Can say that the vectors are “orthogonal”
  - Can combine for even more fun!
    - “Use normalized surface normals in the calculation.”

# Normalize

- Can normalize a vector by dividing it by its magnitude
  - Example: $\dfrac{\mathbf{u}}{|\mathbf{u}|}$
  - Vector has the same direction, but the magnitude will be 1.0
  - Also works with scalars

# *Dot Product*

- Why is the dot product so interesting?

# *Dot Product*

- Why is the dot product so interesting?
  - The dot product of two vectors is related to the cosine of the angle between those vectors
  - Formally: $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\ |\mathbf{v}| \cos\theta$
- We often want to know the angle between two vectors
  - This is the basis of all lighting calculations in 3D graphics!

$$\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|} = \frac{\mathbf{u}}{|\mathbf{u}|} \cdot \frac{\mathbf{v}}{|\mathbf{v}|} = \cos\theta$$

# *Cross Product*

- From Wikipedia:

  [T]he cross product is a binary operation on two vectors in a three-dimensional Euclidean space that results in another vector which is perpendicular to the plane containing the two input vectors.

  - Noted as an $\times$ between two vectors
  - Calculated as:

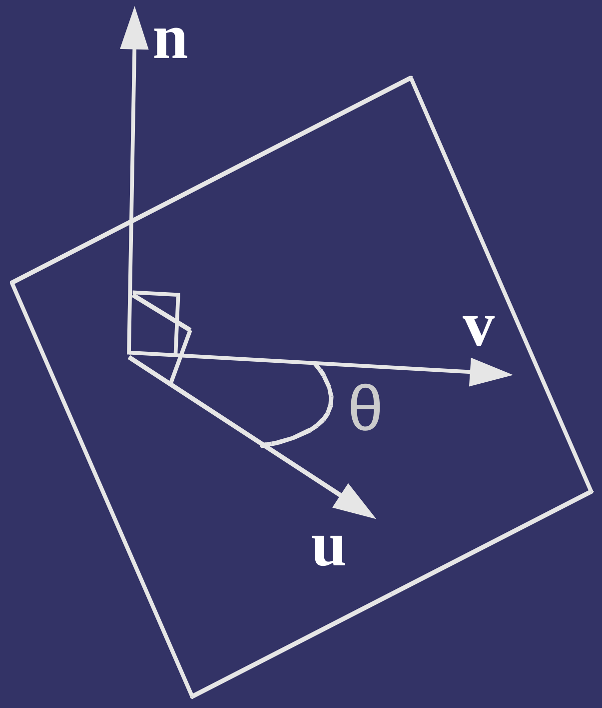
$$\mathbf{a}\times\mathbf{b}=\left[\mathbf{a}_y\mathbf{b}_z-\mathbf{a}_z\mathbf{b}_y \quad \mathbf{a}_z\mathbf{b}_x-\mathbf{a}_x\mathbf{b}_z \quad \mathbf{a}_x\mathbf{b}_y-\mathbf{a}_y\mathbf{b}_x\right]$$

  - Not associative
  - Anti-commutative: If $\mathbf{u}\times\mathbf{v} = \mathbf{w}$, then $\mathbf{v}\times\mathbf{u} = -\mathbf{w}$

[1] From http://en.wikipedia.org/wiki/Cross_product

# *Cross Product*

- Why is the cross product so interesting?
  - Cross product of two vectors results in a new vector that is normal both
  - The cross product of two vectors is related to the sine of the angle between the vectors
    - Formally: $\mathbf{u} \times \mathbf{v} = |\mathbf{u}|\ |\mathbf{v}| \sin\theta\ \mathbf{n}$



# *Matrices*

- Like vectors, but have multiple rows and columns
  - Example: $\begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$
- Add and subtract like you would expect
  - Like vectors, both matrices must be the same size...in both dimensions

# Matrix Multiplication

- Special rules make matrix multiplication different from scalar multiplication
  - **NOT** commutative!  e.g., $\mathbf{M}\times\mathbf{N} \neq \mathbf{N}\times\mathbf{M}$
  - Associative  e.g., $\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$
  - Column count of first matrix must match row count of second matrix
    - If $\mathbf{M}$ is 4-by-3 matrix and $\mathbf{N}$ is a 3-by-1 matrix, we can do $\mathbf{M}\times\mathbf{N}$ but not $\mathbf{N}\times\mathbf{M}$
  - If the source matrices are $n$-by-$m$ and $m$-by-$p$, the re-sulting matrix will be $n$-by-$p$

# Matrix Multiplication

- To calculate an element of the matrix, **C**, resulting from **AB**:

$$\mathbf{C}_{ij} = \Sigma_{r=1}^{n} \mathbf{A}_{ir} \mathbf{B}_{rj}$$
$$= \mathbf{A}_{i,0} \mathbf{B}_{0,j} + \mathbf{A}_{i,1} \mathbf{B}_{1,j} + \mathbf{A}_{i,2} \mathbf{B}_{2,j} + \ldots + \mathbf{A}_{i,n} \mathbf{B}_{n,j}$$

- What does this look like?

# Matrix Multiplication

- To calculate an element of the matrix, $\mathbf{C}$, resulting from $\mathbf{AB}$:

$$\begin{aligned}\mathbf{C}_{ij} &= \Sigma_{r=1}^{n} \mathbf{A}_{ir} \mathbf{B}_{rj} \\ &= \mathbf{A}_{i,0}\mathbf{B}_{0,j} + \mathbf{A}_{i,1}\mathbf{B}_{1,j} + \mathbf{A}_{i,2}\mathbf{B}_{2,j} + \ldots + \mathbf{A}_{i,n}\mathbf{B}_{n,j}\end{aligned}$$

- What does this look like?
  - The dot product of a row of $\mathbf{A}$ with a column of $\mathbf{B}$!
  - This is why the column count of $\mathbf{A}$ must match the row count of $\mathbf{B}$...otherwise the dot product wouldn't work

# *Multiplicative Identity*

- There is a multiplicative identity for matrices

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

  - Just like any other multiplicative identity, $\mathbf{AI} = \mathbf{A}$
  - If you pretend that a scalar is a $1 \times 1$ matrix, this should make sense

# *Transpose*

- Rows become columns and columns become rows
  - Noted with a T in the exponent position (e.g., $\mathbf{M}^T$)
  - Example:

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$$

# *Matrix Multiplication*

- Can rewrite the dot product (inner product) of two row vectors as:

$$s = \mathbf{u}\,\mathbf{v}^{\mathrm{T}}$$

- Can write the *outer product* of two row vectors as:

$$\mathbf{M} = \mathbf{u}^{\mathrm{T}}\,\mathbf{v}$$

  - Notation is $\mathbf{u} \otimes \mathbf{v}$

$$\mathbf{u} \otimes \mathbf{v} = \begin{bmatrix} \mathbf{u}_1\mathbf{v}_1 & \mathbf{u}_1\mathbf{v}_2 & \mathbf{u}_1\mathbf{v}_3 & \ldots & \mathbf{u}_1\mathbf{v}_n \\ \mathbf{u}_2\mathbf{v}_1 & \mathbf{u}_2\mathbf{v}_2 & \mathbf{u}_2\mathbf{v}_3 & \ldots & \mathbf{u}_2\mathbf{v}_n \\ \cdots & \cdots & \ddots & & \cdots \\ \mathbf{u}_m\mathbf{v}_1 & \mathbf{u}_m\mathbf{v}_2 & \mathbf{u}_m\mathbf{v}_3 & \ldots & \mathbf{u}_m\mathbf{v}_n \end{bmatrix}$$

# *Matrix Multiplication*

- Not commutative

$$\mathbf{M} \times \mathbf{N} \neq \mathbf{N} \times \mathbf{M}$$

- But...

$$\mathbf{M} \times \mathbf{N} = \left(\mathbf{N}^T \times \mathbf{M}^T\right)^T$$

- How is this useful?

# *Matrix Multiplication*

- Not commutative

$$\mathbf{M} \times \mathbf{N} \neq \mathbf{N} \times \mathbf{M}$$

- But...

$$\mathbf{M} \times \mathbf{N} = \left(\mathbf{N}^T \times \mathbf{M}^T\right)^T$$

- How is this useful?
  - Assume $v$ is a vector we want to transform by a matrix $M$, but we only have $M^T$ in our program...

$$\mathbf{M} \times \mathbf{v} = \left(\mathbf{v}^T \times \mathbf{M}^T\right)^T$$

  - A vector and its transpose are represented the same way (`vec4` in GLSL), so we don't have to do the transpose of the matrix

# *References*

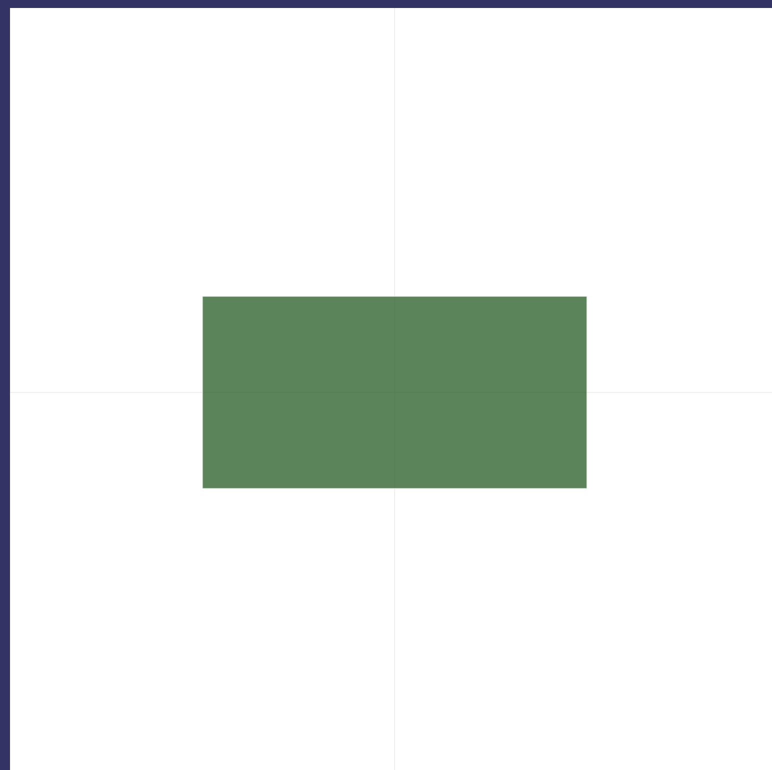
http://en.wikipedia.org/wiki/Matrix_multiplication

http://en.wikipedia.org/wiki/Dot_product

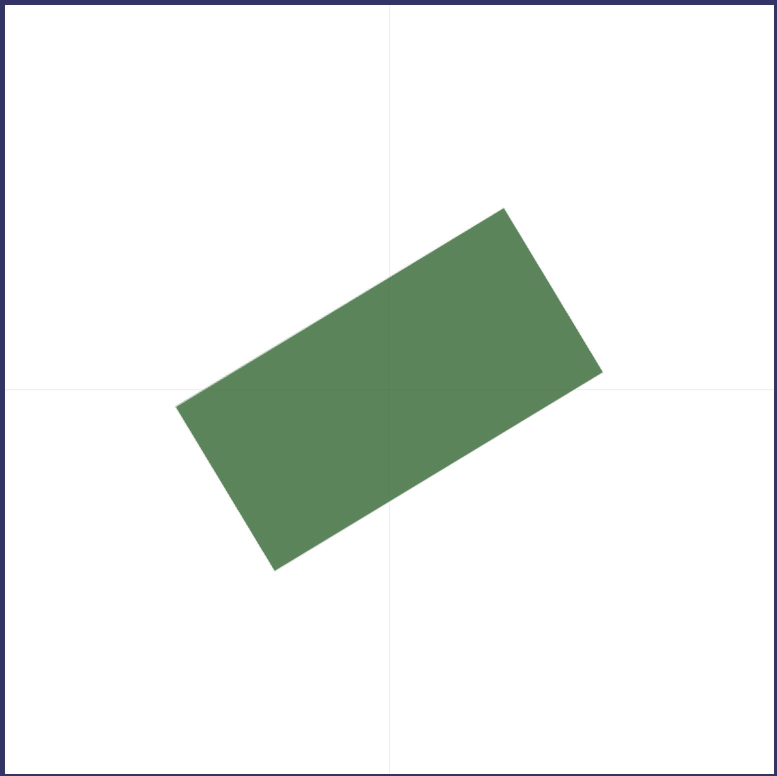
http://en.wikipedia.org/wiki/Cross_product

http://en.wikipedia.org/wiki/Outer_product

# *Rotation*

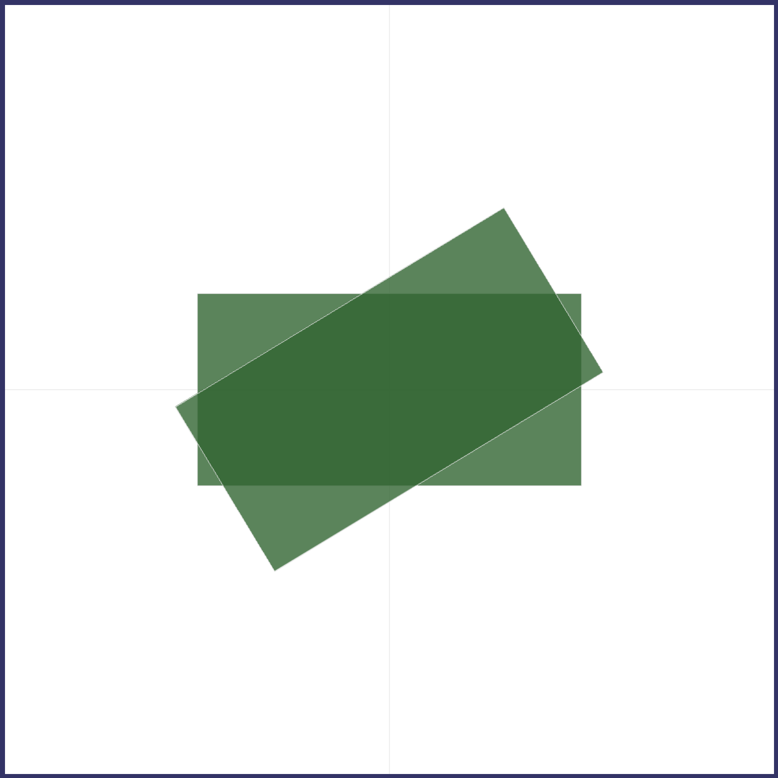


# *Rotation*

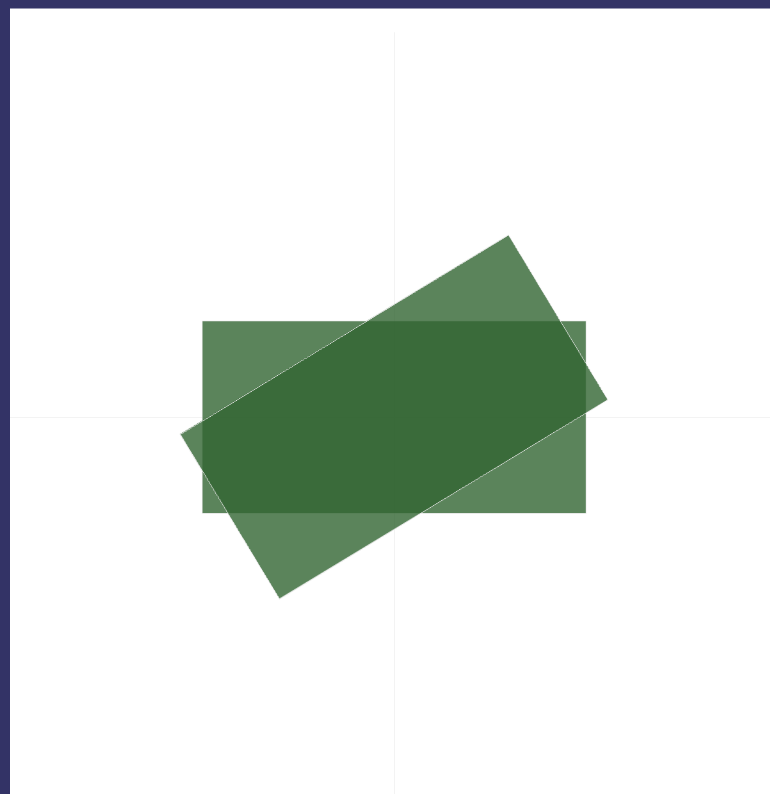


12-October-2011

# *Rotation*



# *Rotation*



$$\mathbf{x}\cos\theta - \mathbf{y}\sin\theta$$
$$\mathbf{x}\sin\theta + \mathbf{y}\cos\theta$$

12-October-2011

# Rotation

$$\begin{aligned} \mathbf{x}' &= \mathbf{x}\cos\theta - \mathbf{y}\sin\theta \\ \mathbf{y}' &= \mathbf{x}\sin\theta + \mathbf{y}\cos\theta \end{aligned}$$

# Rotation

$$\begin{aligned} \mathbf{x}' &= \mathbf{x}\cos\theta - \mathbf{y}\sin\theta \\ \mathbf{y}' &= \mathbf{x}\sin\theta + \mathbf{y}\cos\theta \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} \mathbf{x}' &= \begin{bmatrix}\cos\theta & -\sin\theta\end{bmatrix} \cdot \begin{bmatrix}\mathbf{x} & \mathbf{y}\end{bmatrix} \\ \mathbf{y}' &= \begin{bmatrix}\sin\theta & \cos\theta\end{bmatrix} \cdot \begin{bmatrix}\mathbf{x} & \mathbf{y}\end{bmatrix} \end{aligned}$$

# *Rotation*

$$\begin{aligned}\mathbf{x}' &= \mathbf{x}\cos\theta - \mathbf{y}\sin\theta \\ \mathbf{y}' &= \mathbf{x}\sin\theta + \mathbf{y}\cos\theta\end{aligned}$$

$$\begin{aligned}\mathbf{x}' &= \begin{bmatrix}\cos\theta & -\sin\theta\end{bmatrix} \cdot \begin{bmatrix}\mathbf{x} & \mathbf{y}\end{bmatrix} \\ \mathbf{y}' &= \begin{bmatrix}\sin\theta & \cos\theta\end{bmatrix} \cdot \begin{bmatrix}\mathbf{x} & \mathbf{y}\end{bmatrix}\end{aligned}$$

$$\begin{bmatrix}\mathbf{x}' \\ \mathbf{y}'\end{bmatrix} = \begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\mathbf{x} \\ \mathbf{y}\end{bmatrix}$$

# *Rotation*

- Rotation around the Z-axis
  - If $\theta$ is $0^\circ$, this is the identity matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Rotation around the Y-axis

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Rotation

- Why use the matrix method?
  - We can rotate using 4 multiplies and 2 adds
  - A matrix multiply requires 16 multiplies and 12 adds

$$\begin{aligned} \mathbf{x}' &= \mathbf{x}\cos\theta - \mathbf{y}\sin\theta \\ \mathbf{y}' &= \mathbf{x}\sin\theta + \mathbf{y}\cos\theta \\ \mathbf{z}' &= z \end{aligned}$$

# Rotation

- A series of rotations can be implemented as:

$$\begin{aligned}\mathbf{v}' &= \mathbf{M}_1\mathbf{v}\\ \mathbf{v}'' &= \mathbf{M}_2\mathbf{v}'\\ \mathbf{v}''' &= \mathbf{M}_3\mathbf{v}''\end{aligned}$$

- With substitution:

$$\begin{aligned}\mathbf{v}''' &= \mathbf{M}_3\mathbf{v}''\\ &= \mathbf{M}_3\left(\mathbf{M}_2\mathbf{v}'\right)\\ &= \mathbf{M}_3\left(\mathbf{M}_2\left(\mathbf{M}_1\mathbf{v}\right)\right)\end{aligned}$$

  - Why is this useful?

# *Rotation*

- A series of rotations can be implemented as:

$$\begin{aligned} \mathbf{v}' &= \mathbf{M}_1 \mathbf{v} \\ \mathbf{v}'' &= \mathbf{M}_2 \mathbf{v}' \\ \mathbf{v}''' &= \mathbf{M}_3 \mathbf{v}'' \end{aligned}$$

- With substitution:

$$\begin{aligned} \mathbf{v}''' &= \mathbf{M}_3 \mathbf{v}'' \\ &= \mathbf{M}_3 \left( \mathbf{M}_2 \mathbf{v}' \right) \\ &= \mathbf{M}_3 \left( \mathbf{M}_2 \left( \mathbf{M}_1 \mathbf{v} \right) \right) \\ &= \left( \mathbf{M}_3 \mathbf{M}_2 \mathbf{M}_1 \right) \mathbf{v} \end{aligned}$$

  - Matrix multiplication is associative!

# *Rotation*

- A series of rotations can be implemented as:

$$\begin{aligned} \mathbf{v}' &= \mathbf{M}_1 \mathbf{v} \\ \mathbf{v}'' &= \mathbf{M}_2 \mathbf{v}' \\ \mathbf{v}''' &= \mathbf{M}_3 \mathbf{v}'' \end{aligned}$$

The matrices are composed in the <u>reverse</u> order of how they are applied to the vector!

- With substitution:

$$\begin{aligned} \mathbf{v}''' &= \mathbf{M}_3 \mathbf{v}'' \\ &= \mathbf{M}_3 \left( \mathbf{M}_2 \mathbf{v}' \right) \\ &= \mathbf{M}_3 \left( \mathbf{M}_2 \left( \mathbf{M}_1 \mathbf{v} \right) \right) \\ &= \left( \mathbf{M}_3 \mathbf{M}_2 \mathbf{M}_1 \right) \mathbf{v} \end{aligned}$$

  - Matrix multiplication is associative!

# *Arbitrary Rotation*

- Given a vector, $\mathbf{v}$, and an angle, $\theta$, we can create an arbitrary rotation matrix:

$$\tilde{\mathbf{V}} = \begin{bmatrix} 0 & -\mathbf{v}_z & \mathbf{v}_y & 0 \\ \mathbf{v}_z & 0 & -\mathbf{v}_x & 0 \\ -\mathbf{v}_y & \mathbf{v}_x & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R} = \left(\mathbf{I}\cos\theta\right) - \left(\left(1-\cos\theta\right)\left(\mathbf{v}\otimes\mathbf{v}\right)\right) + \left(\tilde{\mathbf{V}}\sin\theta\right)$$

# Translation

- Points are stored as $\mathbf{p} = [\ x\ y\ z\ 1\ ]$
- Remember the definition of matrix multiplication:

$$
\begin{aligned}
\mathbf{p}_x' &= \mathbf{p}_x\mathbf{M}_{11} + \mathbf{p}_y\mathbf{M}_{12} + \mathbf{p}_z\mathbf{M}_{13} + \mathbf{p}_w\mathbf{M}_{14} \\
\mathbf{p}_y' &= \mathbf{p}_x\mathbf{M}_{21} + \mathbf{p}_y\mathbf{M}_{22} + \mathbf{p}_z\mathbf{M}_{23} + \mathbf{p}_w\mathbf{M}_{24} \\
\mathbf{p}_z' &= \mathbf{p}_x\mathbf{M}_{31} + \mathbf{p}_y\mathbf{M}_{32} + \mathbf{p}_z\mathbf{M}_{33} + \mathbf{p}_w\mathbf{M}_{34} \\
\mathbf{p}_w' &= \mathbf{p}_x\mathbf{M}_{41} + \mathbf{p}_y\mathbf{M}_{42} + \mathbf{p}_z\mathbf{M}_{43} + \mathbf{p}_w\mathbf{M}_{44}
\end{aligned}
$$

- Since $\mathbf{p}_w$ is always 1, the 4th column of the matrix acts as a translation

# *Scaling*

- To scale a vector, multiply each component by a scale factor

$$\mathbf{M} = \begin{bmatrix} \mathbf{s}_x & 0 & 0 & 0 \\ 0 & \mathbf{s}_y & 0 & 0 \\ 0 & 0 & \mathbf{s}_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# *Next week...*

- Quiz #1
  - Will cover material from last week and this week
- More transformations
- Hidden surface removal / occlusion
  - Backface culling
  - Painters algorithm
  - Z-buffer
  - Frustum culling
- Assignment #2, part 1

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