VGP352 – Week 6

- Agenda:
  - Illuminating infinitesimal strands
    - Piles of math leading to the Banks BRDF
    - General “strand” model for anisotropic surfaces
  - Goldman's “fakefur”
  - Implementing BRDFs in real-time
  - Fins-and-shells for fur

Hair

- How do we calculate illumination for an infinitesimal strand or fiber?

Terminology – Codimension

- Definition:

  Given an object of dimension n in a k dimensional space, with $k > n$, the codimension, c, is equal to $k - n$

  - For a surface in 3-space, $n = 2$ and $k = 3$
    - When $c = 1$, we can trivially assign a normal to the object

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  - For a surface in 3-space, $n = 2$ and $k = 3$
    - When $c = 1$, we can trivially assign a normal to the object
  - For a line in 3-space, $n = 1$ and $k = 3$
    - When $c > 1$, things get a little weird...

Terminology – Codimension

- Another way to think of it: The normal has c degrees of freedom
  - For a plane in 3-space, the normal can point in one of two directions (up or down)
    - $k - n = c \Rightarrow 3 - 2 = 1$
  - It's only degree of freedom is its magnitude
    - If we restrict the space to normalized vectors, there are only two possible values

Terminology – Vector Spaces

- Definition:

  A vector space is a mathematical structure formed by a collection of vectors: objects that may be added together and multiplied ("scaled") by numbers, called scalars in this context.[1]

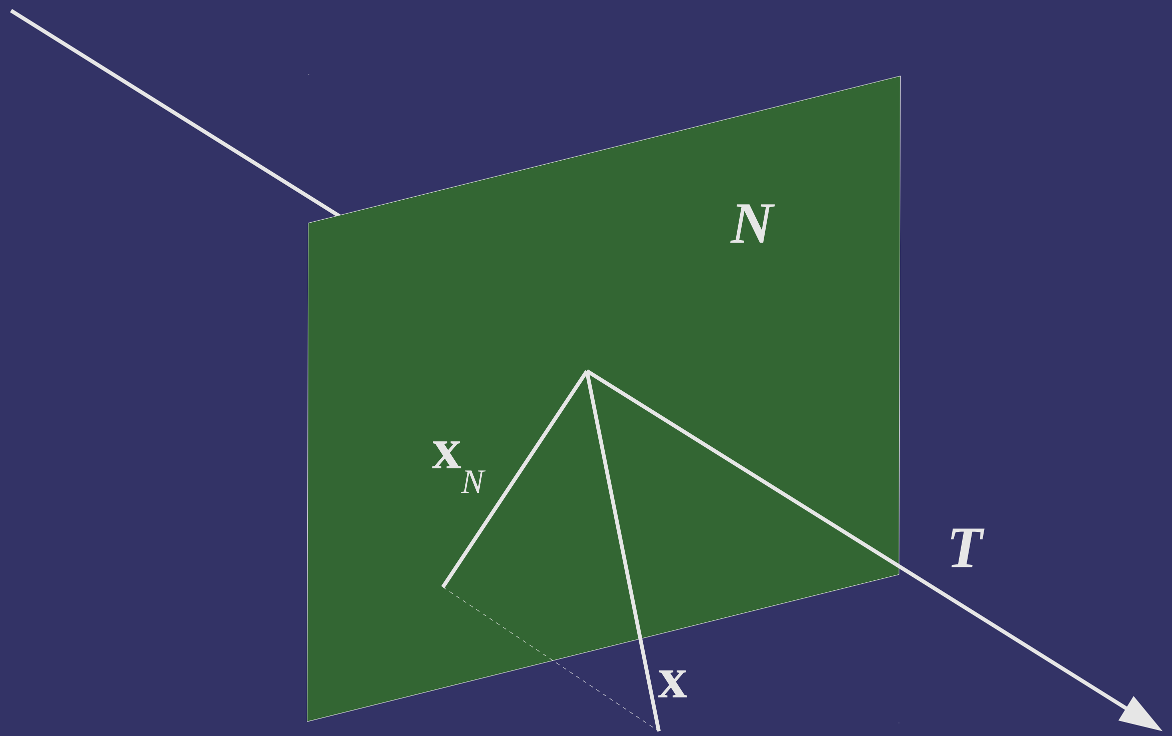
[1]From http://en.wikipedia.org/wiki/Vector_space

Terminology – Vector Spaces

- T is the tangent-space at some point on the object
  - Vector space tangent to the point on the object
    - Specifically, all of the possible tangent vectors at that location
  - Has dimension n (same as the object)
- N is the normal-space at some point on the object
  - Vector space orthogonal to T
    - Specifically, all of the possible normal vectors at that location
  - Has dimension c (codimension of the object)

Terminology – Vector Projections

- $\mathbf{x}_N$ is the projection of vector $\mathbf{x}$ onto N
- $\mathbf{x}_T$ is the projection of vector $\mathbf{x}$ onto T



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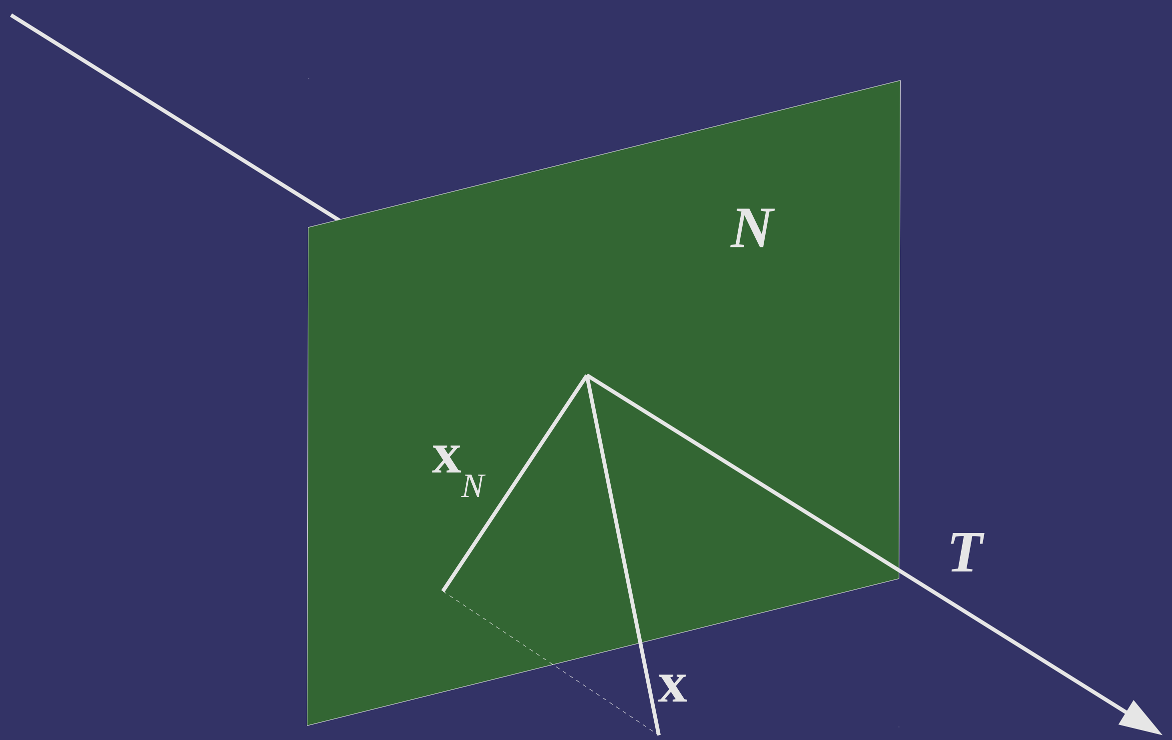
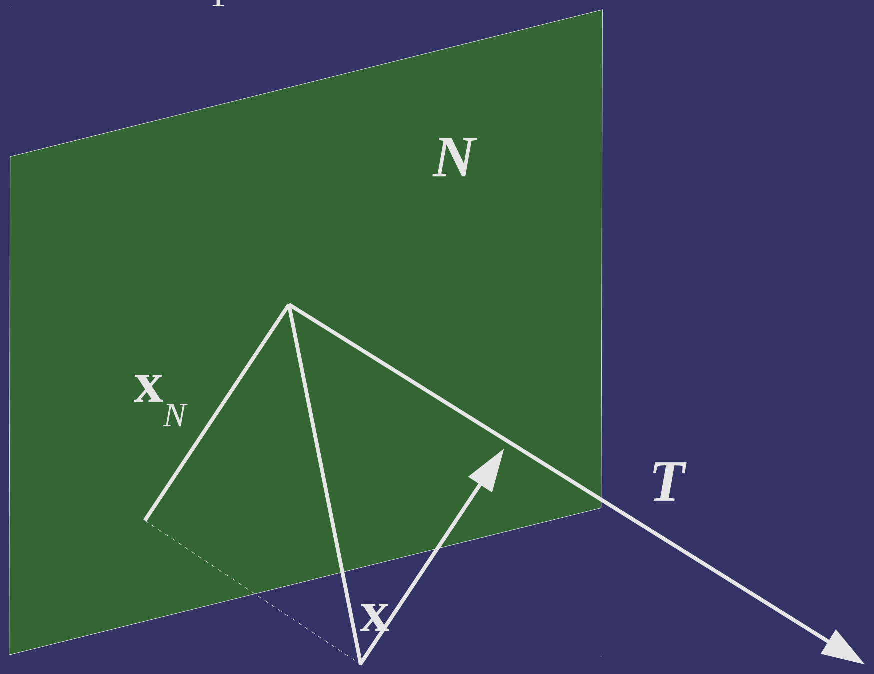
Terminology – Vector Projections

- How do we project a vector onto a plane?



Terminology – Vector Projections

- How do we project a vector onto a plane?
  - Fun facts:
    - $\mathbf{x}_N = \mathbf{x} - \mathbf{x}_T$
    - $|\mathbf{x}_T| = \cos(\mathbf{x}, T) = \mathbf{x} \cdot T \rightarrow \mathbf{x}_T = (\mathbf{x} \cdot T)T$
    - $|\mathbf{x}_N| = \sin(\mathbf{x}, T)$



Diffuse Reflection

- Using this terminology, diffuse reflection can be calculated as:

$$\mathbf{i}_{\mathrm{diffuse}} = \mathbf{k}_{\mathrm{d}} \frac{\cos\left(\mathbf{l}, \mathbf{l}_{\mathrm{N}}\right)}{|\mathbf{l}|\,|\mathbf{l}_{\mathrm{N}}|}$$

- Since N and T are orthogonal, we can rewrite this as:

$$\mathbf{i}_{\mathrm{diffuse}} = \mathbf{k}_{\mathrm{d}} \frac{\sin\left(\mathbf{l}, \mathbf{l}_{\mathrm{T}}\right)}{|\mathbf{l}|\,|\mathbf{l}_{\mathrm{T}}|}$$

Specular Reflection

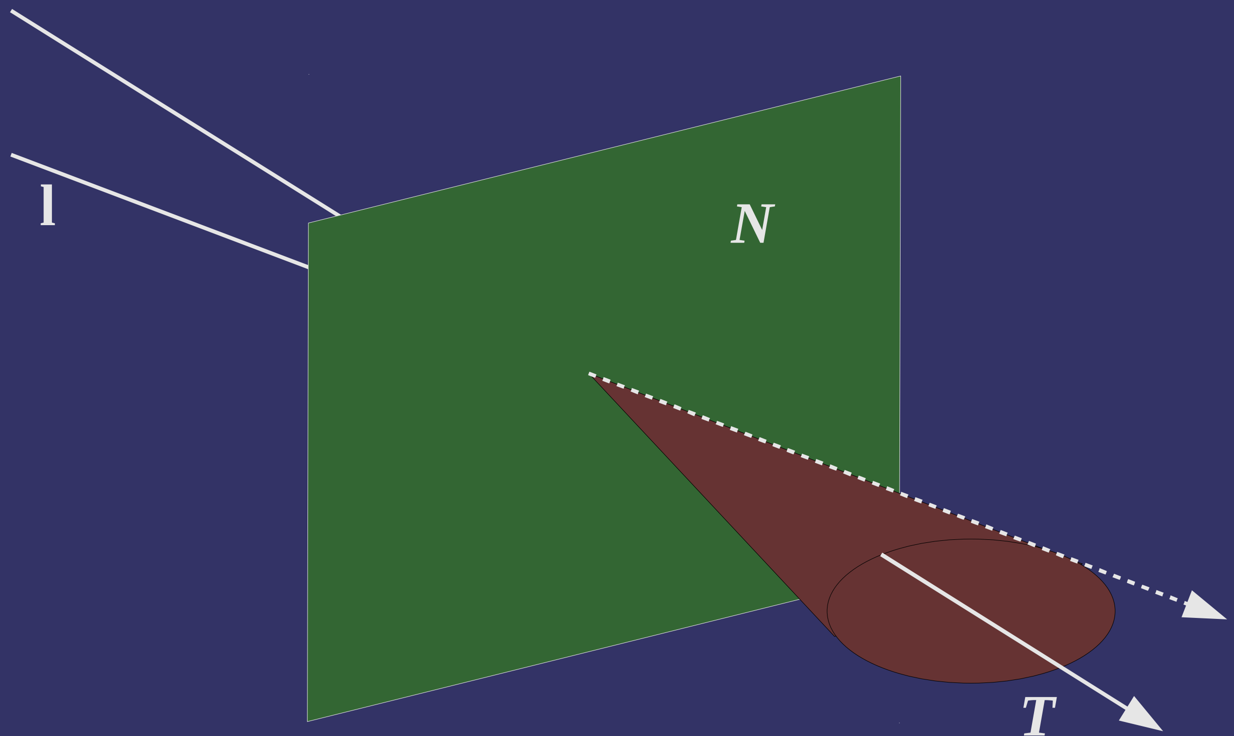
- Phong specular reflection:

$$\mathbf{r} = \mathbf{n} - 2(\mathbf{n} \cdot \mathbf{l})\mathbf{l}$$

$$\mathbf{i}_{\mathrm{specular}} = \mathbf{k}_{\mathrm{s}} \mathbf{i}_{\mathrm{light}} \cos(\mathbf{v}, \mathbf{r})^{s}$$

Specular Reflection

- When $c > 1$, there are infinite possible $\mathbf{n}$ vectors, so there are infinite possible $\mathbf{r}$ vectors



Fermat's Principle

- Fermat's principle says that light travels on the shortest length path
  - This means that $\mathbf{l}$, $\mathbf{l}_N$, and $\mathbf{r}$ are coplanar
  - Skipping a bit of derivation, this means that $\mathbf{l}_N$ is equal to $\mathbf{r}_N$
  - Skipping a bit more derivation, this means that $\mathbf{v} \cdot \mathbf{r}$ can be calculated as:

$$\mathbf{v} \cdot \mathbf{r} = \mathbf{v}_T \cdot \mathbf{l}_T - |\mathbf{v}_N| |\mathbf{l}_N|$$

Specular Reflection

- $\mathbf{v} \cdot \mathbf{r}$ can be calculated as:

$$\mathbf{v} \cdot \mathbf{r} \quad = \quad \mathbf{v}_T \cdot \mathbf{l}_T - |\mathbf{v}_N||\mathbf{l}_N|$$

  – But we don't initially know $\mathbf{v}_T$, $\mathbf{l}_T$, $\mathbf{v}_N$, or $\mathbf{l}_N$

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&= \left((\mathbf{v} \cdot \mathbf{t})\mathbf{t}\right) \cdot \left((\mathbf{l} \cdot \mathbf{t})\mathbf{t}\right) - \sin(\mathbf{v}, \mathbf{t}) \sin(\mathbf{l}, \mathbf{t}) \\
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&= ((\mathbf{v}\cdot\mathbf{t})(\mathbf{l}\cdot\mathbf{t})(\mathbf{t}\cdot\mathbf{t})) - \sin(\mathbf{v},\mathbf{t})\sin(\mathbf{l},\mathbf{t}) \\
&= (\mathbf{v}\cdot\mathbf{t})(\mathbf{l}\cdot\mathbf{t}) - \sin(\mathbf{v},\mathbf{t})\sin(\mathbf{l},\mathbf{t}) \\
&= (\mathbf{v}\cdot\mathbf{t})(\mathbf{l}\cdot\mathbf{t}) - \left(\sqrt{1-\cos(\mathbf{v},\mathbf{t})^2}\sqrt{1-\cos(\mathbf{l},\mathbf{t})^2}\right)
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Specular Reflection

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\end{aligned}$$

Inherited Self-Shadowing

- When c = 1, the object has at most 2 sides
  - One side of the surface “self-shadows” the other
  - We get that calculation for free from $\mathbf{n} \cdot \mathbf{l}$

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- Consider a surface with a 2D tangent space, T, and a 1D vector field, V
  - If T is used to calculate the illumination, $\mathbf{n}_{surface} \cdot \mathbf{l}$ works
  - If V is used to calculate the illumination, there is no unique $\mathbf{n}$ to use

Think of bristles on a surface

Inherited Self-Shadowing

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  - If V is used to calculate the illumination, there is no unique $\mathbf{n}$ to use
  - If V is used to calculate the illumination, it can *inherit* $\mathbf{n}_{\text{surface}} \cdot \mathbf{l}$ from T

$$\mathbf{i}_{\text{conditioned}} = \max\left(\mathbf{n} \cdot \mathbf{l}, 0\right)\left(\mathbf{i}_{\text{diffuse}} + \mathbf{i}_{\text{specular}}\right)$$

Vector Field Shadowing

- This shadows the vector field from the surface
  - If the vectors lie outside the surface (e.g., fur) the vector field can obviously shadow itself and the surface
- Input light energy is attenuated by:

$$d = h / \sin(\mathbf{t}, \mathbf{l})$$
$$\mathbf{i}_{\text{atten}} = \mathbf{i}_{\text{source}} (1 - \rho)^{d}$$

  - h is the distance from the surface
  - ρ is a property of the fur
    - The paper uses ρ = 0.02

Strand Based Anisotropic Lighting

- Why limit the use of this lighting model to individual strands?
  - We can treat many types of anisotropic surfaces as a collection of many strands... and apply the same lighting technique!

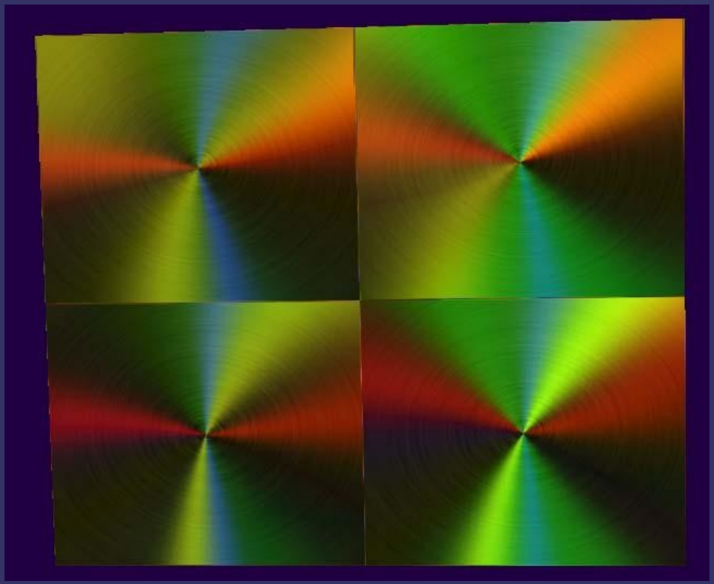

Image from ShaderX, Wordware Publishing, Inc.

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Banks, D. C. 1994. Illumination in diverse codimensions. In *Proceedings of the 21st Annual Conference on Computer Graphics and interactive Techniques SIGGRAPH '94*. ACM, New York, NY, 327-334. http://lmi.bwh.harvard.edu/~banks/

Isidoro, John and Brennan, Chris. "Per-Pixel Strand Based Anisotropic Lighting" in Engel, Wolfgang F. (editor) ShaderX, Wordware Publishing, Inc., May 2002. http://developer.amd.com/documentation/reading/pages/ShaderX.aspx

fakefur

- Developed by Dan Goldman at ILM
  - A *much* faster version of the “realfur” algorithm used at ILM for close-up shots

fakefur

- Makes several simplifying assumptions:
  - Geometry of individual hairs is not visible
  - Hairs are truncated cones
  - The length of each cone is much greater than the radius of either end
    - Can't be used to render 5 o'clock shadow!
  - Radius of the base is greater than the radius of the other end
  - All hairs in an area have identical geometry

Algorithm Overview

- Compute average hair geometry in sample area
- For each light:
  - Compute hair-over-hair shadow attenuation
  - Compute reflected luminance of hair
  - Compute hair-over-skin shadow attenuation
  - Compute reflected luminance of skin
  - Compute hair / skin visibility ratio
  - Blend skin and hair reflected luminances using hair / skin visibility ratio
- Sum per-light calculated values

Illumination Function

$$
\begin{aligned}
\Psi_{\text{diffuse}} &= \mathbf{k}_{\mathrm{d}} \sin(\mathbf{t},\mathbf{l}) \\
\Psi_{\text{specular}} &= \mathbf{k}_{\mathrm{s}} \left((\mathbf{t}\cdot\mathbf{l})(\mathbf{t}\cdot\mathbf{v}) + \sin(\mathbf{t},\mathbf{l}) \sin(\mathbf{t},\mathbf{v}) \right)^{p} \\
\Psi_{\text{hair}} &= \Psi_{\text{diffuse}} + \Psi_{\text{specular}}
\end{aligned}
$$

- Why is sine used instead of cosine?

Illumination Function

$$\Psi_{\text{diffuse}} = \mathbf{k}_{\text{d}} \sin(\mathbf{t},\mathbf{l})$$

$$\Psi_{\text{specular}} = \mathbf{k}_{\text{s}} \left(\boxed{(\mathbf{t}\cdot\mathbf{l})(\mathbf{t}\cdot\mathbf{v})+\sin(\mathbf{t},\mathbf{l})\sin(\mathbf{t},\mathbf{v})}\right)^{p}$$

$$\Psi_{\text{hair}} = \Psi_{\text{diffuse}} + \Psi_{\text{specular}}$$

- Why is sine used instead of cosine?
  - We treat the hair as having dimension = 1
    - There are infinite possible normals
    - There is only one tangent

This should look familiar!

Illumination Function

$$\Psi_{\text{diffuse}} = \mathbf{k}_{\text{d}} \sin(\mathbf{t}, \mathbf{l})$$

$$\Psi_{\text{specular}} = \mathbf{k}_{\text{s}} \left((\mathbf{t} \cdot \mathbf{l})(\mathbf{t} \cdot \mathbf{v}) + \sin(\mathbf{t}, \mathbf{l}) \sin(\mathbf{t}, \mathbf{v}) \right)^{p}$$

$$\Psi_{\text{hair}} = \Psi_{\text{diffuse}} + \Psi_{\text{specular}}$$

- What's wrong here?

Illumination Function

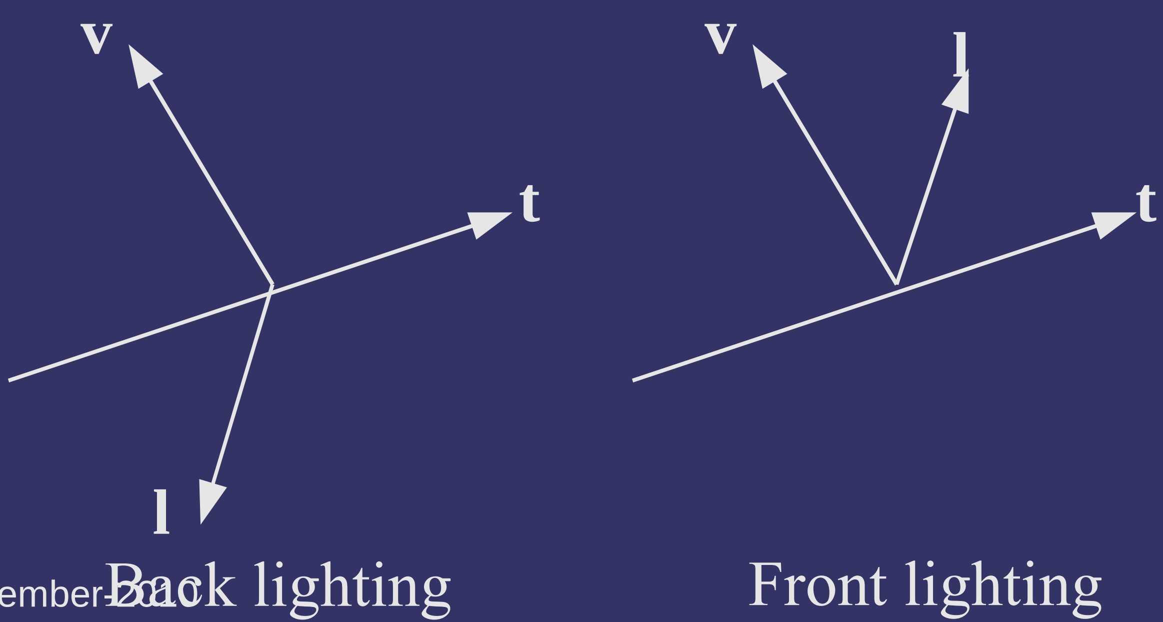
$$\begin{aligned}\Psi_{\text{diffuse}} &= \mathbf{k}_{\text{d}} \sin(\mathbf{t},\mathbf{l}) \\ \Psi_{\text{specular}} &= \mathbf{k}_{\text{s}} \left((\mathbf{t}\cdot\mathbf{l})(\mathbf{t}\cdot\mathbf{v}) + \sin(\mathbf{t},\mathbf{l})\sin(\mathbf{t},\mathbf{v})\right)^{p} \\ \Psi_{\text{hair}} &= \Psi_{\text{diffuse}} + \Psi_{\text{specular}}\end{aligned}$$

- What's wrong here?
  - Lacks directionality
    - Hairs are fully lit even if $\mathbf{l}$ is opposite $\mathbf{v}$
  - Fix this by adding some new attenuation factors

Relative Directionality

$$\kappa = \frac{(\boldsymbol{t} \times \boldsymbol{l}) \cdot (\boldsymbol{t} \times \boldsymbol{v})}{|\boldsymbol{t} \times \boldsymbol{l}||\boldsymbol{t} \times \boldsymbol{v}|}$$

- $\kappa > 0$ when $\mathbf{l}$ and $\mathbf{v}$ are on the same side of the hair (frontlighting)
- $\kappa < 0$ when $\mathbf{l}$ and $\mathbf{v}$ are on opposite sides of the hair (backlighting)



Directional Attenuation Factor

$$f_{dir} = \frac{1+\kappa}{2}\rho_{reflect} + \frac{1-\kappa}{2}\rho_{transmit}$$

- $\rho_{reflect}$ and $\rho_{transmit}$ are parameters of the hair on the range [0, 1]
- White and gray hairs have $\rho_{reflect}$ and $\rho_{transmit}$ equal or nearly equal
- Colored hairs have $\rho_{reflect} > \rho_{transmit}$

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- White and gray hairs have $\rho_{reflect}$ and $\rho_{transmit}$ equal or nearly equal
- Colored hairs have $\rho_{reflect} > \rho_{transmit}$
- Unless you're a kitten...

Self-Shadowing

- Controlled by a second attenuation factor and 3 new parameters:

$$f_{\text{surface}} = 1 + \rho_{\text{surface}}\left(\text{smoothstep}\left(\mathbf{n}\cdot\mathbf{l}, \theta_{\min}, \theta_{\max}\right) - 1\right)$$

  - $\rho_{surface}$ controls the amount of self-shadowing
  - θ_{min} is the minimum angle where shadowing occurs
  - θ_{max} is the angle beyond which there is total occlusion

Fur Opacity

- How much of the surface *below* the fur can be seen through the fur?
  - Contributing factors:
    - Hair density: More hairs result in more occlusion
    - Hair size: Larger (thicker) hairs individually occlude more
    - Hair orientation: Hairs “laying down” occlude more than hairs on end
      - Orientation relative to both the viewer and the underlying surface are factors

On end

Laying down

Fur Opacity

- How much of the surface *below* the fur can be seen through the fur?

$$\alpha_{\mathrm{f}} = 1 - \frac{1}{e^{d\, a_{\mathrm{h}}\, \mathrm{g}(\mathbf{v},\mathbf{t},\mathbf{n})}}$$

$$\mathrm{g}(\mathbf{v},\mathbf{t},\mathbf{n}) = \frac{\sin(\mathbf{v},\mathbf{t})}{\mathbf{v}\cdot\mathbf{n}}$$

$$a_{\mathrm{h}} = l_{\mathrm{hair}}(r_{\mathrm{base}} + r_{\mathrm{top}})/2$$

  - d is the local hair density
  - a_h is the projection of the surface area of a hair onto the view plane

Putting It All Together

- Put the attenuation factors together with the opacity and skin color:

$$\begin{aligned}
\Psi_{\text{hair}} &= f_{\text{dir}} f_{\text{surface}} \left(\Psi_{\text{diffuse}} + \Psi_{\text{specular}}\right) \\
\lambda_{\text{skin}} &= \mathbf{k}_{\text{light}} \left(1 - \alpha_{\text{f}}\right) \Psi_{\text{skin}} \\
\lambda_{\text{hair}} &= \mathbf{k}_{\text{light}} \left(1 - \frac{\alpha_{\text{f}}}{2}\right) \Psi_{\text{hair}} \\
f &= \alpha_{\text{f}} \lambda_{\text{hair}} + \left(1 - \alpha_{\text{f}}\right) \lambda_{\text{skin}}
\end{aligned}$$

  - Ψ_{skin} is calculated by some other means

Implementing BRDFs in Real-Time

- BRDF formulations assume integration over all incoming light in the positive hemisphere
  - Clearly impractical for real-time rendering!
  - Not very practical for off-line rendering either...
- Four high-level strategies:
  - Only implement point lights
    - Direction implementation
    - Factorization
  - Reflection map based pre-filtering / pre-calculation
  - Monte Carlo sampling
  - Deferred shading based techniques

Point Light Direct Implementation

- Use $\mathbf{l}$ for ω_i and $\mathbf{v}$ for ω_o and directly implement the math
  - We already do this for Phong lighting
  - More complex lighting equations can be prohibitively expensive
  - Since we're *not* integrating over the hemisphere, multiply the BRDF by π

Factorization

- Expensive equations can be factored into sums or products of functions of fewer variables
  - Each input vector (i.e., $\mathbf{v}$, $\mathbf{l}$, $\mathbf{h}$, $\mathbf{n}$, etc.) or dot-product of vectors becomes an input to one function
  - Each function is stored in some sort of texture
  - This technique works really well for sampled BRDFs
- Using two textures, the Poulin-Fournie anisotropic satin BRDF can be implemented as:

$$\alpha\, p(\mathbf{v})\, q(\mathbf{h})\, p(\mathbf{l})$$

  - $p()$ and $q()$ represent texture look-ups and α is a special scaling factor

Factorization

- Remember the Banks BRDF for strands:

$$\mathbf{v}\cdot\mathbf{r}=(\mathbf{v}\cdot\mathbf{t})(\mathbf{l}\cdot\mathbf{t})-\left(\sqrt{1-(\mathbf{v}\cdot\mathbf{t})^2}\sqrt{1-(\mathbf{l}\cdot\mathbf{t})^2}\right)$$

Factorization

- Remember the Banks BRDF for strands:

$$\mathbf{v}\cdot\mathbf{r}=(\mathbf{v}\cdot\mathbf{t})(\mathbf{l}\cdot\mathbf{t})-\left(\sqrt{1-(\mathbf{v}\cdot\mathbf{t})^2}\sqrt{1-(\mathbf{l}\cdot\mathbf{t})^2}\right)$$

  - Note that $\mathbf{v}\cdot\mathbf{r}$ is a function of two dot-products
  - Store all possible values of $\mathbf{v}\cdot\mathbf{r}$ in a 2D texture and sample this texture using $\mathbf{v}\cdot\mathbf{t}$ and $\mathbf{l}\cdot\mathbf{t}$

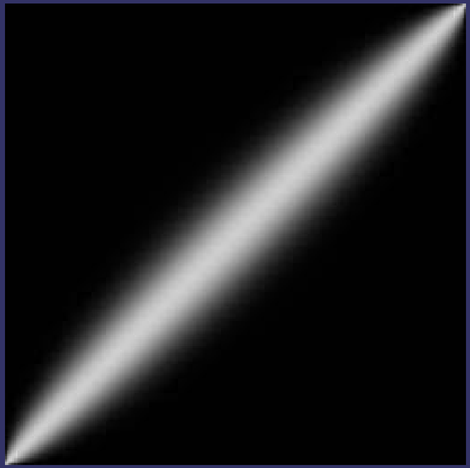

Image from ShaderX, Wordware Publishing, Inc.

References

University of Waterloo Factored BRDF Repository: http://www.cgl.uwaterloo.ca/Projects/rendering/Shading/database.html

Michael D. McCool, Jason Ang, Anis Ahmad, *Homomorphic Factorization of BRDFs for High-Performance Rendering*, SIGGAPH 2001, August 12-17, 2001. http://www.cgl.uwaterloo.ca/Projects/rendering/Papers/

Reflection Maps

- Reflection maps present additional challenges
  - Decent lighting require multiple samples
- As with the Phong lighting model, reflection maps can be pre-filtered using complex BRDFs
  - Doesn't work well with dynamic env. maps
  - Doesn't work *at all* with aniostropic BRDFs
    - The ideal reflection vector isn't enough information!

Grid Sampling

- Sample the reflection map at multiple, predetermined locations, use the sample vectors and the sampled values in the lighting equation

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  - Might not sample the most important vectors for the lighting equation
    - For most equations, samples closer $\mathbf{r}$ are more important

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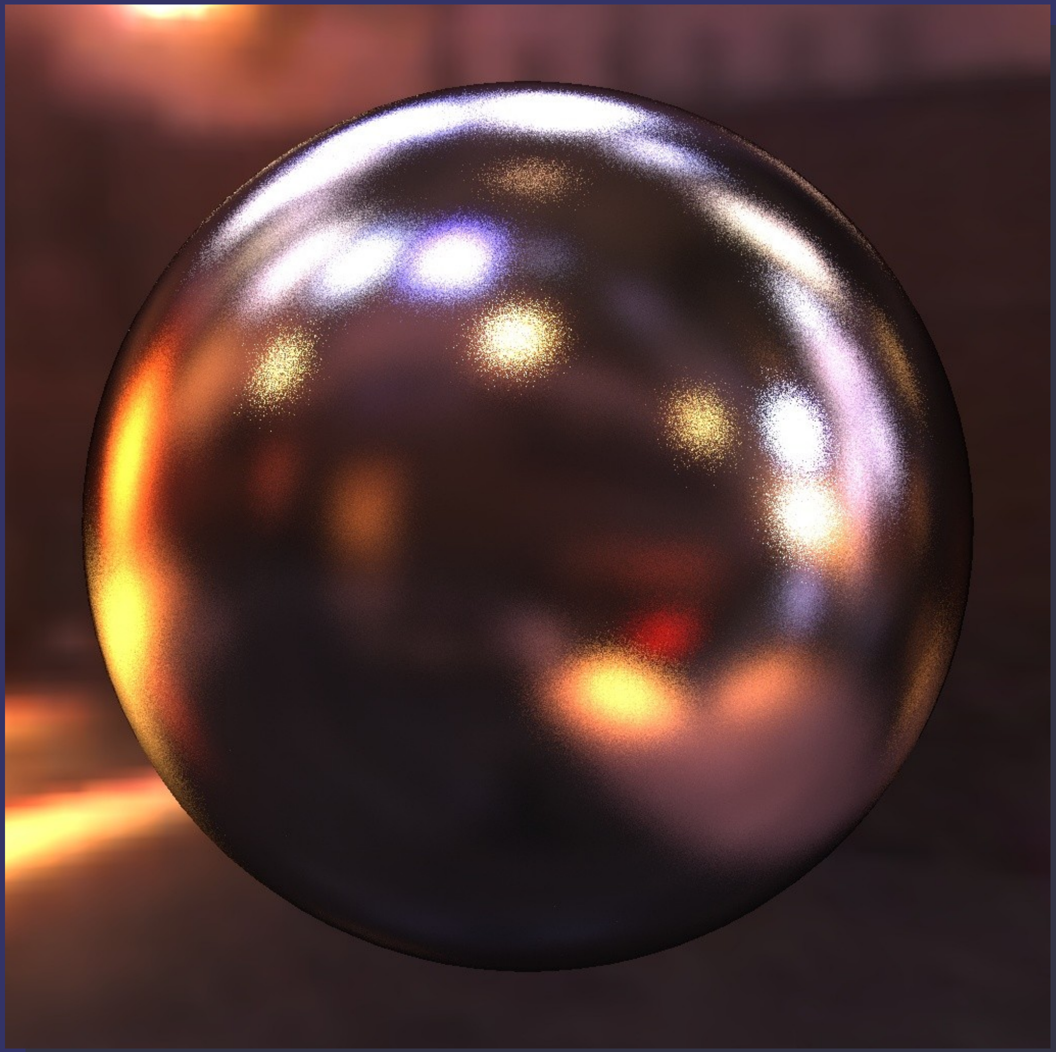
- Sample the reflection map at multiple, predetermined locations, use the sample vectors and the sampled values in the lighting equation
  - Might not sample the most important vectors for the lighting equation
    - For most equations, samples closer $\mathbf{r}$ are more important
  - Might not sample the most important vectors for the reflection map
    - If most of the refelction map is dark with just a few bright spots, those bright spots are more important
    - This problem is especially difficult to solve

Monte Carlo Integration

- Instead of sampling at regular intervals, sample at pseudo-random locations

Monte Carlo Integration



Images rendered with 40 samples per-fragment

Images from “Real-time Shading with Filtered Importance Sampling”
http://graphics.cs.ucf.edu/gpusampling/filter_is_intel.ppt
10-November-2010

Monte Carlo Integration

- Instead of sampling at regular intervals, sample at pseudo-random locations
  - Must sample many locations to eliminate noise
    - Where “many” may mean *thousands*
  - Or determine the random locations with a BRDF-dependent probability density function (PDF)
    - Several of the papers from this term include a PDF for the BRDF
    - Still several problems:
      - Generating good random numbers on the GPU is hard
      - Requires quite a few samples
        - Colbert and Křivánek found that around 40 looks good

Monte Carlo Integration

- Monte Carlo estimator for a BRDF:

$$\mathrm{L}(\mathbf{v}) \approx \frac{1}{n} \sum_{k=1}^{n} \frac{\mathrm{L_i}(\mathbf{u}_k) f(\mathbf{u}_k, \mathbf{v}) \cos\theta_{\mathbf{u}_k}}{p(\mathbf{u}_k, \mathbf{v})}$$

  - p is the PDF
  - $\mathbf{u}_k$ is a random light direction generated based on the PDF
    - Typically generate a uniform random value and remap it based on the PDF

Monte Carlo Integration

Images rendered with 40 samples per-fragment

Images from “Real-time Shading with Filtered Importance Sampling”
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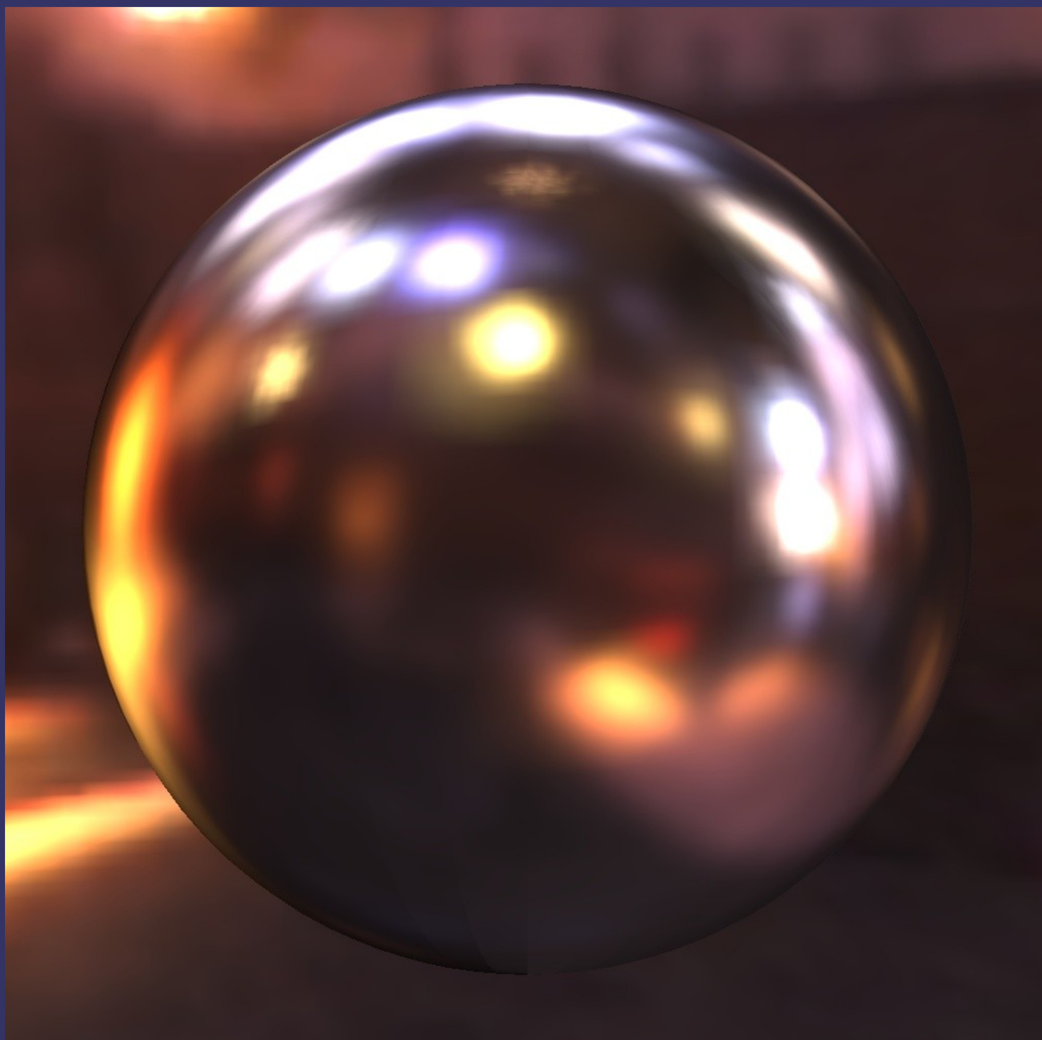
Monte Carlo Integration

- Deterministic importance sampling causes unacceptable aliasing effects
  - Less important samples (i.e., less probable) are low-weighted individual points
  - Observe that neighbors of less probably samples are unlikely to be sampled
    - Allow those neighbors to contribute by using the PDF to select a mipmap level in the reflection map
    - Higher mipmap levels average larger regions into a single texel
    - Cube maps have weird distortion away from the axes, so a different type of reflection map should be used
      - The paper suggests dual-paraboloid maps

Monte Carlo Integration



Images rendered with 40 samples per-fragment

Images from “Real-time Shading with Filtered Importance Sampling”
http://graphics.cs.ucf.edu/gpusampling/filter_is_intel.ppt
10-November-2010

References

Colbert, M. and Křivánek, J. 2007. “Real-time shading with filtered importance sampling.” In *ACM SIGGRAPH 2007 Sketches* (San Diego, California, August 05 - 09, 2007). SIGGRAPH '07. ACM, New York, NY, 71. http://graphics.cs.ucf.edu/gpusampling/

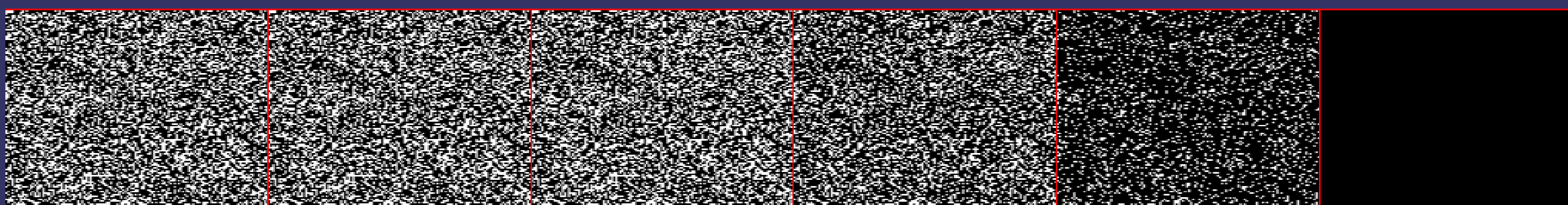
http://en.wikipedia.org/wiki/Monte_Carlo_integration

Volumetric Fur

- Close-up, fur appears as a volumetric effect
- Kajika and Kay presented an algorithm at SIGGRAPH '89 implementing fur via 3D textures
  - Volumetric textures are *very* memory intensive
  - Kajika and Kay's model involves several computationally expensive steps
- Not practical for real-time
  - There has to be a different way!

Shells and Fins

- Instead of a 3D texture, fur can be implemented with a “stack” of 2D textures
  - Each layer in the stack represents the fur at a different depth



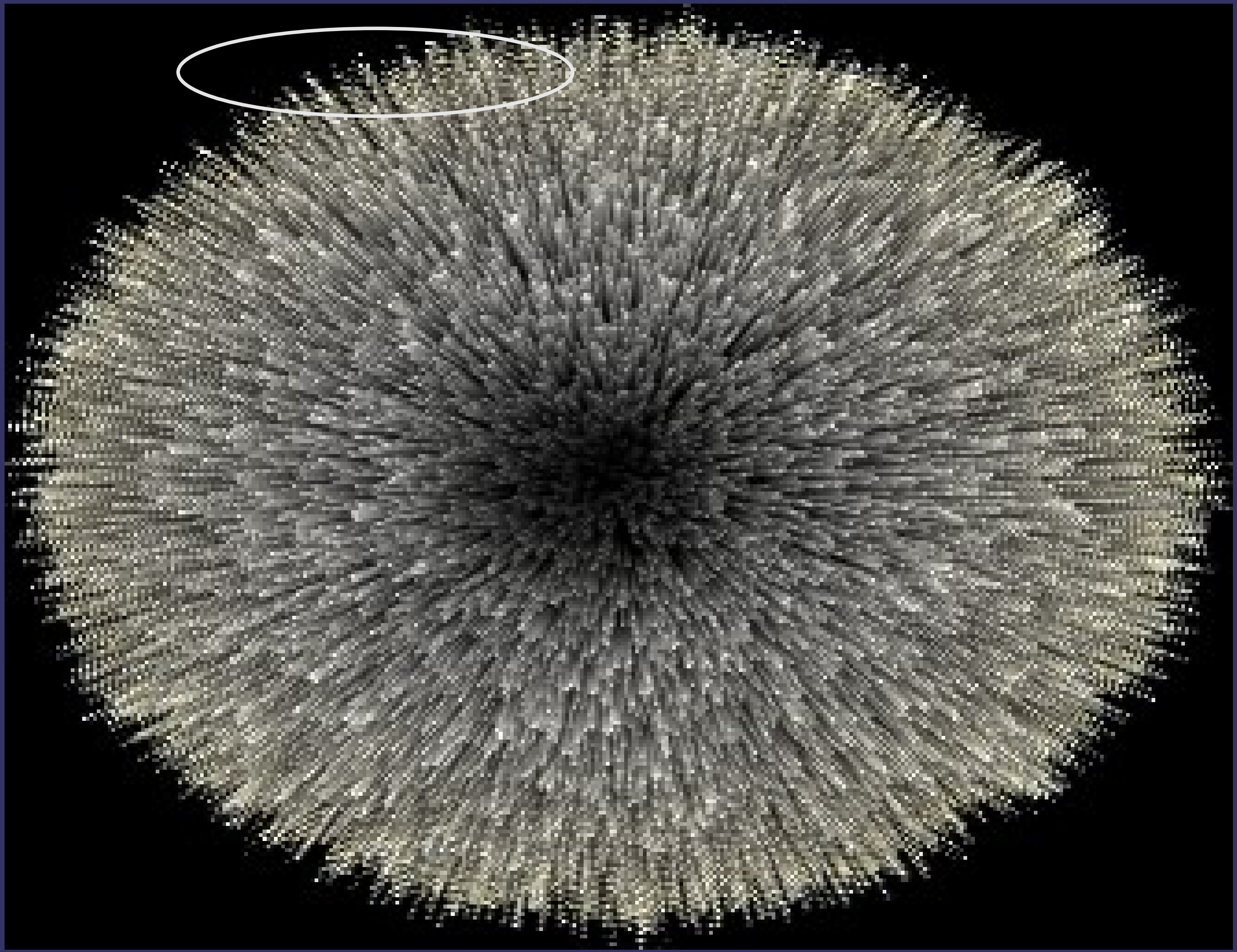
  - Draw each layer in a progressively larger “shell” around the original object geometry

Shells and Fins

- Drawing loop:
  - Draw base object with inner-most (call it level 0) fur texture
    - Disable alpha blending
    - Enable z-testing
    - Enable z-writing
  - Draw base geometry moved out some small step along the normals
    - Enable alpha blending
    - Enable z-testing
    - Disable z-writing

Shells and Fins

- *But* this looks bad along the silhouette



Shells and Fins

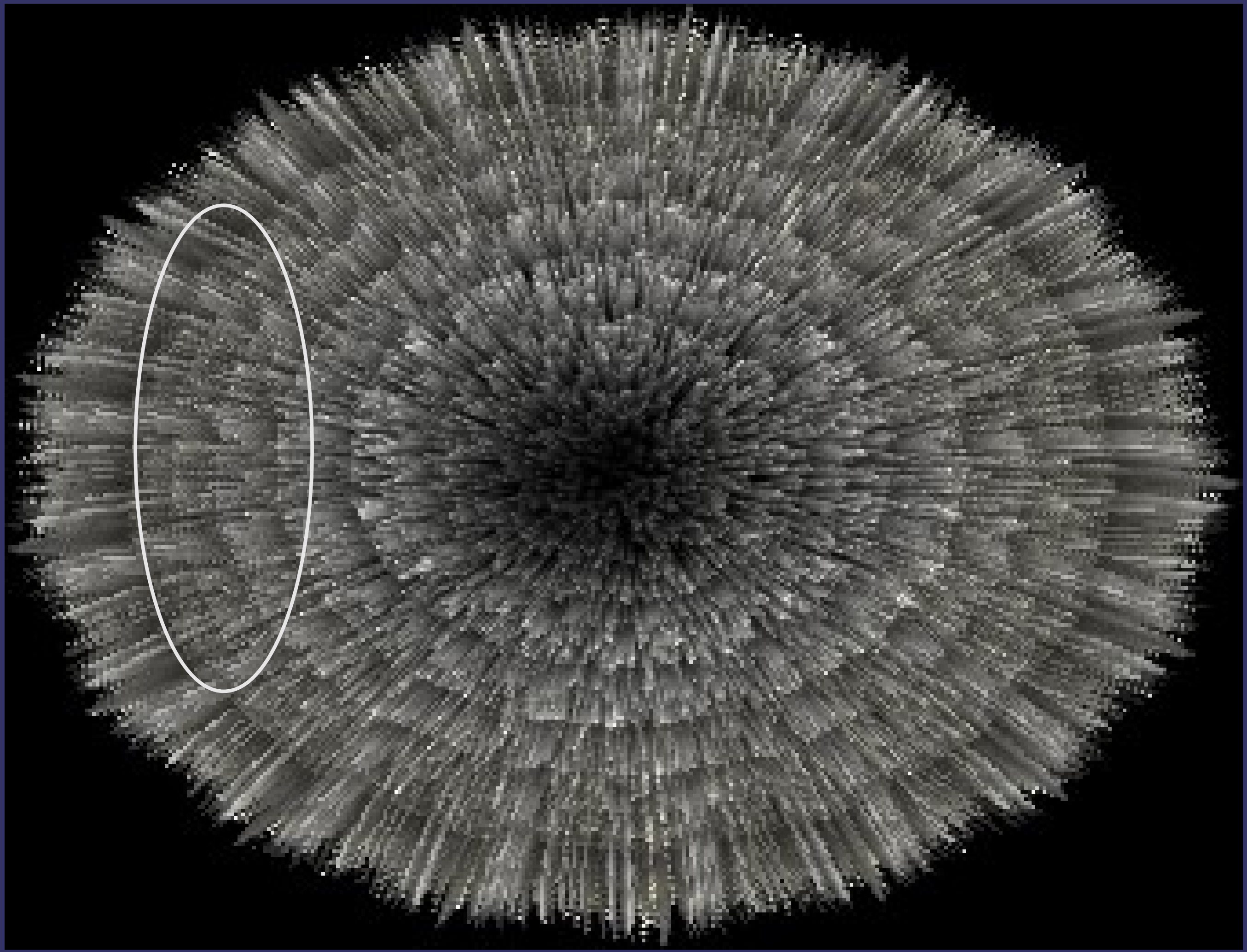
- Add fin geometry to each polygon
  - Create fin textures to look like side-on view of fur
  - Draw fin after drawing all shells
    - Enable alpha blending
    - Enable z-testing
    - Disable z-writing

Shells and Fins

- Generate fin geometry in the vertex shader:
  - Draw each vertex *twice*
    - Once with w = 0
    - Once with w = 1
  - Use the w value to determine whether or not to extrude the vertex in the normal direction
  - Draw the vertices as two triangles:
    - One with vertices 0, 1, 1
    - The other with vertices 1, 0, 0

Shells and Fins

- *But* this looks bad in non-silhouette areas



Shells and Fins

- Gradually blend in fins as they approach the silhouette

$$\alpha_{\text{fin}} = \max\left(0, 2\left|\cos\left(\mathbf{v}, \mathbf{n}_{\text{fin}}\right)\right| - 1\right)$$

- We don't really have a fin normal...what to do?

Shells and Fins

- Gradually blend in fins as they approach the silhouette

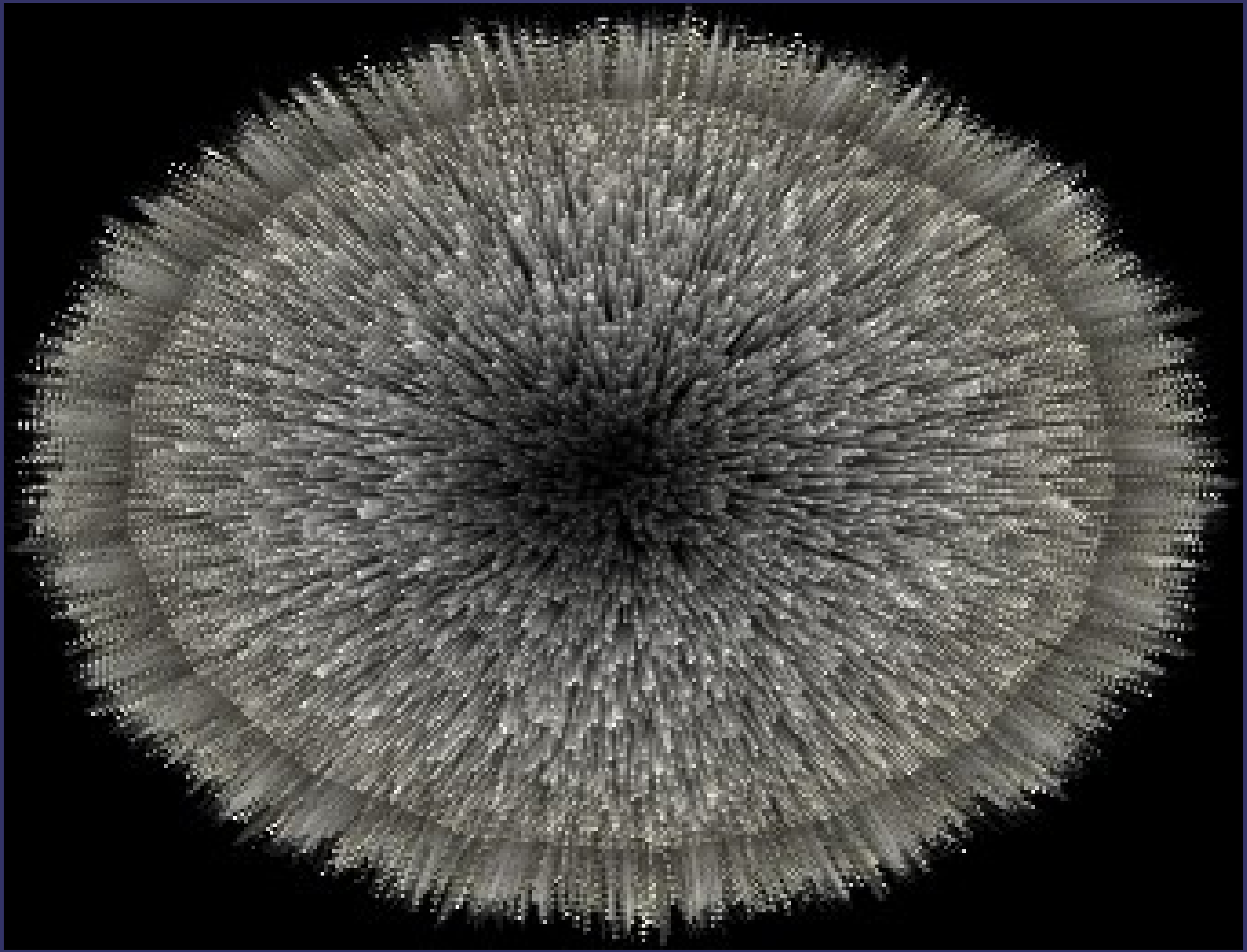
$$\alpha_{\text{fin}} = \max\left(0, 2\left|\cos\left(\mathbf{v}, \mathbf{n}_{\text{fin}}\right)\right| - 1\right)$$

- We don't really have a fin normal...what to do?
  - The surface's normal is the fin's tangent

$$\alpha_{\text{fin}} = \max\left(0, 2\left|\sin\left(\mathbf{v}, \mathbf{n}_{\text{surface}}\right)\right| - 1\right)$$

Shells and Fins

- Alpha blended fins



Lighting Shells and Fins

- Use the surface normal as the direction of the hair

$$\mathbf{k} = \mathbf{k}_{\mathrm{d}} \sin(\mathbf{n}_{\mathrm{surface}}, \mathbf{l})^{p_{\mathrm{d}}} + \mathbf{k}_{\mathrm{s}} \sin(\mathbf{n}_{\mathrm{surface}}, \mathbf{h})^{p_{\mathrm{s}}}$$

  - p_d and p_s are diffuse and specular exponents
  - Similar to Goldman's fakefur lighting model
- A little trig-identity love gets us:

$$\begin{aligned}\mathbf{k} &= \mathbf{k}_{\mathrm{d}} \left(1 - \cos^2(\mathbf{n}_{\mathrm{surface}}, \mathbf{l})\right)^{p_{\mathrm{d}}/2} + \mathbf{k}_{\mathrm{s}} \left(1 - \cos^2(\mathbf{n}_{\mathrm{surface}}, \mathbf{h})\right)^{p_{\mathrm{s}}/2} \\ &= \mathbf{k}_{\mathrm{d}} \left(1 - (\mathbf{n}_{\mathrm{surface}} \cdot \mathbf{l})^2\right)^{p_{\mathrm{d}}/2} + \mathbf{k}_{\mathrm{s}} \left(1 - (\mathbf{n}_{\mathrm{surface}} \cdot \mathbf{h})^2\right)^{p_{\mathrm{s}}/2}\end{aligned}$$

Lighting Shells and Fins

- No shadowing happens!
  - Fur near the skin is occluded by the fur above it
  - Add a shadowing term to falloff to a minimum value linearly with the distance from the outermost shell

$$s = \frac{d\left(1 - s_{min}\right)}{d_{max}} + s_{min}$$

  - d is the current shell distance
    - $d = 0$ is the shell closest to the skin
  - d_{max} is the total number of shells
  - s_{min} is the minimum amount of light reaching the bottom layer

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Thorough overview of the various real-time fur methods.

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Lake, A. and Kuah, K.. *Real-Time Fur Rendering For Short Haired Creatures*. 2006. http://softwarecommunity.intel.com/articles/eng/2597.htm

Morris, N. CS6610 Final Project. December 2005. http://www.cs.utah.edu/classes/cs5610/projects-2005/morris/

Next week...

- Quiz #3
- Non-photorealistic rendering
  - Cel (toon) shading
  - Silhouette edge rendering
  - Technical illustration
- Begin post-processing / image space effects

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