VGP352 – Week 5

- Agenda:
  - Anisotropic reflection
    - Ward BRDF
    - Ashikhmin BRDF
  - Metals
    - The skin effect
    - Lafortune BRDF

3-November-2010

Anisotropy Refresher

> Anisotropy...is the property of being directionally dependent, as opposed to isotropy, which means homogeneity in all directions. It can be defined as a difference in a physical property (absorbance, refractive index, density, etc.) for some material when measured along different axes. An example is the light coming through a polarizing lens.

- We saw this last term with filter areas for texture sampling

Anisotropic Reflection

- What does anisotropy mean for lighting and reflections?
  - Some materials reflect light differently depending on the orientation of the material w.r.t. the light and viewer

Anisotropic Reflection


Statue near 707 NW 11th Ave, Portland OR


Floor in the Fourth Avenue Building, Portland State University

Anisotropic Reflection

- What causes anisotropic reflection?
  - Think about the micro-facet theory of surfaces

Anisotropic Reflection

- What causes anisotropic reflection?
  - Think about the micro-facet theory of surfaces
  - The distribution of normals is random, but the distribution depends on the orientation

Anisotropic Reflection

- What additional information is needed to implement an anisotropic normal distribution function?
  - Our current lighting models use $\mathbf{h}$, derived from $\mathbf{n}$, $\mathbf{l}$, and $\mathbf{v}$
  - This gives no information for the relative orientation of the surface vs. the light and viewer

Anisotropic Reflection

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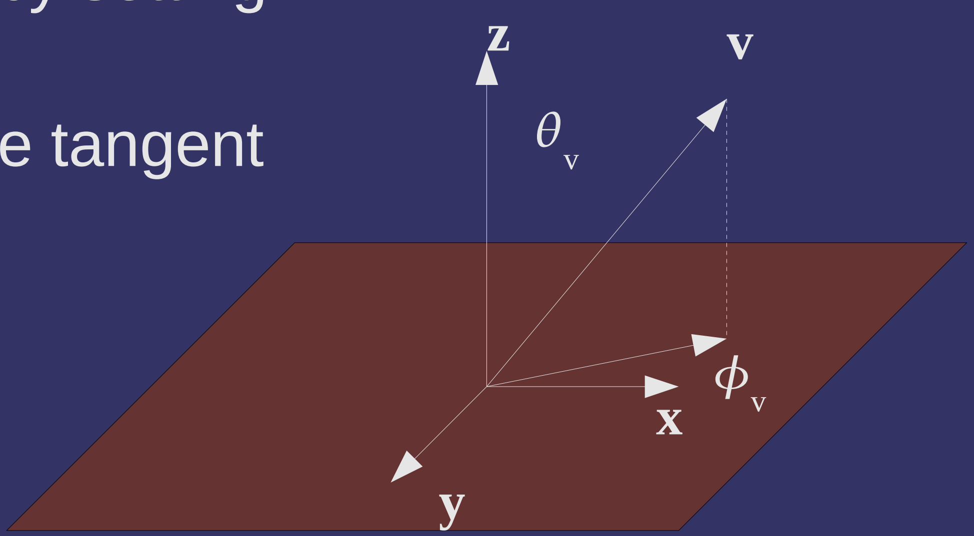
  - Our current lighting models use $\mathbf{h}$, derived from $\mathbf{n}$, $\mathbf{l}$, and $\mathbf{v}$
  - This gives no information for the relative orientation of the surface vs. the light and viewer
- The surface tangent!
  - If $\mathbf{v}'$ is the projection of $\mathbf{v}$ onto the plane containing $\mathbf{t}$ and $\mathbf{b}$, $arccos(\mathbf{v}' \cdot \mathbf{t})$ is the relative orientation angle

Ward's Anisotropic Model

- Map **n**, **t**, and **b** to the **z**, **x**, and **y** axes
  - $\theta_{\mathbf{v}}$ is the angle between the vector and the Z-axis
    - We can get this from the usual dot-products
  - $\phi_{\mathbf{v}}$ is the angle between the vector and the X-axis
    - Project **v** into the X/Y plane by setting Z=0 and re-normalizing
    - Take the dot-product with the tangent



Ward's Anisotropic Model

$$f(\omega_i, \omega_o) = \frac{\mathbf{k}_d}{\pi} + \frac{\mathbf{k}_s}{4\pi\alpha_x\alpha_y\sqrt{\cos\theta_i\cos\theta_o}} e^{-\tan^2\theta_{\mathbf{h}}\left(\frac{\cos^2\phi_{\mathbf{h}}}{\alpha_x^2}+\frac{\sin^2\phi_{\mathbf{h}}}{\alpha_y^2}\right)}$$

- α_x and α_y control the width of the highlight in the two principal directions
  - $\alpha_x = \alpha_y$ the reflection is isotropic
  - $\sin^2\theta = 1 - \cos^2\theta$
  - $\tan^2\theta = \sin^2\theta / \cos^2\theta$

Ward's Anisotropic Model

$$f(\omega_i, \omega_o) = \frac{\mathbf{k}_d}{\pi} + \frac{\mathbf{k}_s}{4\pi\alpha_x\alpha_y\sqrt{\cos\theta_i\cos\theta_o}} e^{-\tan^2\theta_{\mathbf{h}}\left(\frac{\cos^2\phi_{\mathbf{h}}}{\alpha_x^2} + \frac{\sin^2\phi_{\mathbf{h}}}{\alpha_y^2}\right)}$$

- Essentially an elliptical version of the Gaussian distribution
- 1 / ($4\pi\alpha_x\alpha_y$) is a semi-magic normalization factor that "is accurate as long as α is not much greater than 0.2, when the surface becomes mostly diffuse."

Ward's Anisotropic Model

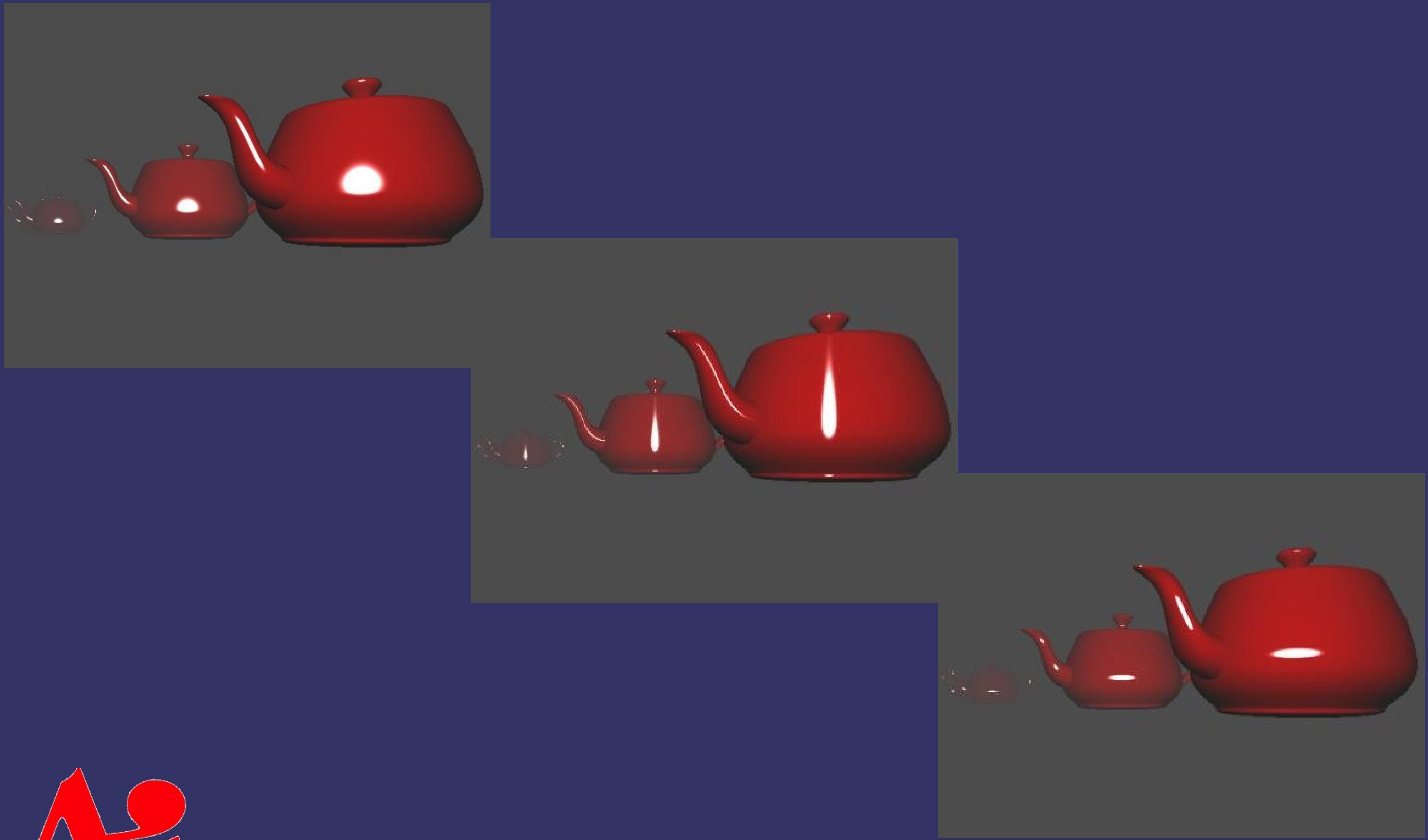
$$f(\omega_i, \omega_o) = \frac{\mathbf{k}_d}{\pi} + \frac{\mathbf{k}_s}{4\pi\alpha_x\alpha_y\sqrt{\cos\theta_i\cos\theta_o}} e^{-\tan^2\theta_{\mathbf{h}}\left(\frac{\cos^2\phi_{\mathbf{h}}}{\alpha_x^2} + \frac{\sin^2\phi_{\mathbf{h}}}{\alpha_y^2}\right)}$$

- Ward presents an approximation that is cheaper to compute, but Schlick found the direct vector implementation to be both exact and faster still:

$$f(\omega_i, \omega_o) = \frac{\mathbf{k}_d}{\pi} + \frac{\mathbf{k}_s}{4\pi\alpha_x\alpha_y\sqrt{(\mathbf{n}\cdot\omega_i)(\mathbf{n}\cdot\omega_o)}} e^{-\frac{\left(\frac{\mathbf{h}\cdot\mathbf{t}}{\alpha_x}\right)^2 + \left(\frac{\mathbf{h}\cdot\mathbf{b}}{\alpha_y}\right)^2}{(\mathbf{h}\cdot\mathbf{n})^2}}$$

- Note: Because a squared dot-product of **h** appears in the numerator and denominator, we don't need to normalize **h**

Ward's Anisotropic Model



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Ashikhmin Model

$$f_s(\omega_i,\omega_o)=\frac{\sqrt{(n_x+1)(n_y+1)}}{8\pi}\frac{(\hat{\mathbf{n}}\cdot\mathbf{h})^{n_x\cos^2\phi_{\mathbf{h}}+n_y\sin^2\phi_{\mathbf{h}}}}{(\mathbf{h}\cdot\omega)\max((\hat{\mathbf{n}}\cdot\omega_i),(\hat{\mathbf{n}}\cdot\omega_o))}\mathrm{F}(\omega\cdot\mathbf{h})$$

- Most of the notation is the same as on the previous slides
  - This differs from the notation in Ashikhmin's paper
- n_x and n_y are Phong-like exponents that control the shape of the specular lobe
  - Roughly analogous to α_x and α_y in Ward's model
- $\mathrm{F}(\theta)$ is the Fresnel term

Ashikhmin Model

$$f_d(\omega_i, \omega_o) = \frac{28\,K_d}{23\pi}(1 - F(0°))\left(1 - \left(1 - \frac{\mathbf{n}\cdot\omega_i}{2}\right)^5\right)\left(1 - \left(1 - \frac{\mathbf{n}\cdot\omega_o}{2}\right)^5\right)$$

- The strange constant factor is "designed to ensure energy conservation."

Anisotropic Reflections

- Both models allow control of specular highlight relative to some coordinate space
  - What coordinate space?
  - How can the orientation of the highlight be controlled?

Anisotropic Reflections

- Both models allow control of specular highlight relative to some coordinate space
  - What coordinate space?
  - How can the orientation of the highlight be controlled?
- We're in surface space!
  - Rotate **t** and **b** by an angle
    - This is **t** and **b** in tangent space
  - Or use a “tangent map”
  - Or use procedural tangents
  - Or...

References

Neil Blevins, “Anisotropic Reflections.” June 19th, 2002. http://www.neilblevins.com/cg_education/aniso_ref/aniso_ref.htm

Ashikhmin, M., Shirley, P. "An Anisotropic Phong BRDF Model" Journal of Graphics Tools, v.5, no. 2 (2000), pp.25-32. http://www.cs.utah.edu/~michael/brdfs/

Ward, G. J. 1992. Measuring and modeling anisotropic reflection. *SIGGRAPH Computer Graphics* 26, 2 (July 1992), 265-272. http://www.cs.virginia.edu/~gfx/Courses/2006/DataDriven/bib/appearance/ward92.pdf

Walter, B. Notes on the Ward BRDF. *Technical report PCG-05-06*, Program of Computer Graphics, Cornell University, April 2005. http://www.graphics.cornell.edu/pubs/2005/Wal05.html

E.M. Waves in Conductors

- Electromagnetic waves in conductors cause free electrons in the material to oscilate
  - The frequency of this oscillation is proportional to the frequency of the electromagnetic wave
  - These oscillations create eddy currents inside the material
  - These eddy currents force the primary current very near the surface
  - The change in current density w.r.t. change of depth is known as the *skin effect*

E.M. Waves in Conductors

- Higher frequency waves cause the current to be limited to thinner and thinner skins on the material
  - A 1GHz wave in copper is restricted to ~0.5mm
  - A 60Hz wave in copper is restricted to ~10mm
  - Note: I'm trading a lot of physics here for a lot of hand waving!

E.M. Waves in Conductors

- What does this have to do with lighting?!?

E.M. Waves in Conductors

- What does this have to do with lighting?!?
  - Light is “just” an electromagnetic wave
  - Visible light is ~400THz - ~700THz
    - THz is tera-Hz or 1,000GHz
- As a result, light cannot penetrate deeply into metals
  - Diffuse reflection in dielectrics is primarily caused subsurface scattering
  - Lacking this, metal doesn't have a traditional diffuse reflection component
    - Cook & Torrance pointed this out as well

Metals

- Two main components to metallic reflection:
  - A mostly pure specular component
    - A la Phong or Blinn
  - A *directional* diffuse component
    - Diffuse in the sense that the reflected color is the color of the material
- None of our current models have a directional diffuse component

Lafortune BRDF

- Remember Phong:

$$\mathbf{k} = \mathbf{k}_s (\mathbf{v} \cdot \mathbf{r})^s \mathbf{i}_s$$

  - $\mathbf{r}$ is the ideal reflection vector
  - Calculation using vectors:

$$\mathbf{r} = 2(\mathbf{n} \cdot \mathbf{l})\mathbf{n} - \mathbf{l}$$

  - Calculation using the Householder matrix:

$$\mathbf{r} = \mathbf{l}^T (2\mathbf{n}\mathbf{n}^T - \mathbf{I}) = \mathbf{l}^T \mathbf{M}$$

Lafortune BRDF

- If we're in surface space, what is $\mathbf{M}$?

$$\mathbf{r} = \mathbf{l}^T (2\mathbf{n}\mathbf{n}^T - \mathbf{I}) = \mathbf{l}^T \mathbf{M}$$

Lafortune BRDF

- If we're in surface space, what is $\mathbf{M}$?

$$\mathbf{r} = \mathbf{l}^T \left(2\,\mathbf{n}\,\mathbf{n}^T - \mathbf{I}\right) = \mathbf{l}^T \mathbf{M}$$

  - Remember: $\mathbf{n} = \{\ 0, 0, 1\ \}$

$$\begin{aligned}
\mathbf{M} &= 2\,\mathbf{n}\,\mathbf{n}^T - \mathbf{I} \\
&= 2\{0,0,1\}\{0,0,1\}^T - \mathbf{I} \\
&= 2\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \mathbf{I} \\
&= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

Lafortune BRDF

- What if we could replace $\mathbf{M}$ with some other matrix?

Lafortune BRDF

- What if we could replace $\mathbf{M}$ with some other matrix?
  - We could move the specular lobe!
  - The new matrix must be symmetric ($\mathbf{M} = \mathbf{M}^T$) or it will violate Helmoltz Reciprocity
  - It turns out that almost all cases except very unusual types of anisotropy, $\mathbf{M}$ is also diagonal
    - $\mathbf{c}_x = \mathbf{c}_y$ is also typical

$$\mathbf{M} = \begin{bmatrix} \mathbf{c}_x & 0 & 0 \\ 0 & \mathbf{c}_y & 0 \\ 0 & 0 & \mathbf{c}_z \end{bmatrix}$$

Lafortune BRDF

- We can rearrange the math a bit:

$$\mathbf{k} = \mathbf{k}_s(\mathbf{r}\cdot\mathbf{v})^s\,\mathbf{i}_s$$

$$= \mathbf{k}_s((\mathbf{l}^T\mathbf{M})\cdot\mathbf{v})^s\,\mathbf{i}_s \qquad \mathbf{r} = \mathbf{l}^T\mathbf{M}$$

$$= \mathbf{k}_s((\mathbf{M}^T\mathbf{l})\cdot\mathbf{v})^s\,\mathbf{i}_s$$

$$= \mathbf{k}_s((\mathbf{M}\mathbf{l})\cdot\mathbf{v})^s\,\mathbf{i}_s \qquad \mathbf{M} = \mathbf{M}^T$$

$$= \mathbf{k}_s((\mathbf{c}*\mathbf{l})\cdot\mathbf{v})^s\,\mathbf{i}_s$$

$$= \mathbf{k}_s(\mathbf{c}_x\mathbf{l}_x\mathbf{v}_x + \mathbf{c}_y\mathbf{l}_y\mathbf{v}_y + \mathbf{c}_z\mathbf{l}_z\mathbf{v}_z)^s\,\mathbf{i}_s$$

- What if we could fit measured data to a series of cosine lobes?

$$\mathbf{k} = \sum_i \mathbf{k}_s(\mathbf{c}_{x,i}\mathbf{l}_x\mathbf{v}_x + \mathbf{c}_{y,i}\mathbf{l}_y\mathbf{v}_y + \mathbf{c}_{z,i}\mathbf{l}_z\mathbf{v}_z)^{s_i}\,\mathbf{i}_s$$

Lafortune BRDF

- What does the data look like?
  - For matte steel:

	C_{xy}	C_z	s
Lobe 1, red	-1.11954	1.01272	15.8708
Lobe 1, green	-1.11845	1.01469	15.6489
Lobe 1, blue	-1.11999	1.01942	15.4571
Lobe 2, red	-1.05334	0.69541	111.267
Lobe 2, green	-1.06409	0.662178	88.9222
Lobe 2, blue	-1.08378	0.626672	65.2179
Lobe 3, red	-1.01684	1.00132	180.181
Lobe 3, green	-1.01635	1.00112	184.152
Lobe 3, blue	-1.01529	1.00108	195.773

References

Lafortune, E. P., Foo, S., Torrance, K. E., and Greenberg, D. P. 1997. Non-linear approximation of reflectance functions. In *Proceedings of the 24th Annual Conference on Computer Graphics and interactive Techniques International Conference on Computer Graphics and Interactive Techniques*. ACM Press/Addison-Wesley Publishing Co., New York, NY, 117-126. http://www.graphics.cornell.edu/pubs/1997/LFTG97.html

http://en.wikipedia.org/wiki/Skin_depth

Next week...

- Fur and hair
  - Two final BRDFs
    - Grand unifying theory of anisotropic BRDFs
    - BRDFs for hair
  - Fins and shells

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