VGP352 – Week 5

- Agenda:
  - Anisotropic reflection
    - Ward BRDF
    - Ashikhmin BRDF
  - Metals
    - The skin effect
    - Lafortune BRDF

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Anisotropy Refresher

> Anisotropy...is the property of being directionally dependent, as opposed to isotropy, which means homogeneity in all directions. It can be defined as a difference in a physical property (absorbance, refractive index, density, etc.) for some material when measured along different axes. An example is the light coming through a polarizing lens.

- We saw this last term with filter areas for texture sampling

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Anisotropic Reflection

- What does anisotropy mean for lighting and reflections?
  - Some materials reflect light differently depending on the orientation of the material w.r.t. the light and viewer

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Anisotropic Reflection

Statue near 707 NW 11th Ave, Portland OR

Floor in the Fourth Avenue Building, Portland State University

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Anisotropic Reflection

- What causes anisotropic reflection?
  - Think about the micro-facet theory of surfaces

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Anisotropic Reflection

- What causes anisotropic reflection?
  - Think about the micro-facet theory of surfaces
  - The distribution of normals is random, but the distribution depends on the orientation

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Anisotropic Reflection

- What additional information is needed to implement an anisotropic normal distribution function?
  - Our current lighting models use $\mathbf{h}$, derived from $\mathbf{n}$, $\mathbf{l}$, and $\mathbf{v}$
  - This gives no information for the relative orientation of the surface vs. the light and viewer

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Anisotropic Reflection

- What additional information is needed to implement an anisotropic normal distribution function?
  - Our current lighting models use $\mathbf{h}$, derived from $\mathbf{n}$, $\mathbf{l}$, and $\mathbf{v}$
  - This gives no information for the relative orientation of the surface vs. the light and viewer
- The surface tangent!
  - If $\mathbf{v}'$ is the projection of $\mathbf{v}$ onto the plane containing $\mathbf{t}$ and $\mathbf{b}$, $arccos(\mathbf{v}' \cdot \mathbf{t})$ is the relative orientation angle

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Ward's Anisotropic Model

- Map $\mathbf{n}$, $\mathbf{t}$, and $\mathbf{b}$ to the $\mathbf{z}$, $\mathbf{x}$, and $\mathbf{y}$ axes
  - $\theta_{\mathbf{v}}$ is the angle between the vector and the Z-axis
    - We can get this from the usual dot-products
  - $\phi_{\mathbf{v}}$ is the angle between the vector and the X-axis
    - Project $\mathbf{v}$ into the X/Y plane by setting Z=0 and re-normalizing
    - Take the dot-product with the tangent

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Ward's Anisotropic Model

$$f(\omega_i, \omega_o) = \frac{\mathbf{k}_d}{\pi} + \frac{\mathbf{k}_s}{4\pi\alpha_x\alpha_y\sqrt{\cos\theta_i\cos\theta_o}} e^{-\tan^2\theta_{\mathbf{h}}\left(\frac{\cos^2\phi_{\mathbf{h}}}{\alpha_x^2} + \frac{\sin^2\phi_{\mathbf{h}}}{\alpha_y^2}\right)}$$

- α_x and α_y control the width of the highlight in the two principal directions
  - $\alpha_x = \alpha_y$ the reflection is isotropic
  - $\tan^2\theta = (1 - \cos^2\theta) / \cos^2\theta$
  - $\sin^2\theta = 1 - \cos^2\theta$

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Ward's Anisotropic Model

$$f(\omega_i, \omega_o) = \frac{\mathbf{k}_d}{\pi} + \frac{\mathbf{k}_s}{4\pi\alpha_x\alpha_y\sqrt{\cos\theta_i\cos\theta_o}} e^{-\tan^2\theta_{\mathbf{h}}\left(\frac{\cos^2\phi_{\mathbf{h}}}{\alpha_x^2}+\frac{\sin^2\phi_{\mathbf{h}}}{\alpha_y^2}\right)}$$

- Essentially an elliptical version of the Gaussian distribution
- 1 / ($4\pi\alpha_x\alpha_y$) is a semi-magic normalization factor that "is accurate as long as α is not much greater than 0.2, when the surface becomes mostly diffuse."

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Ward's Anisotropic Model

$$f(\omega_i, \omega_o) = \frac{\mathbf{k}_d}{\pi} + \frac{\mathbf{k}_s}{4\pi\alpha_x\alpha_y\sqrt{\cos\theta_i\cos\theta_o}} e^{-\tan^2\theta_{\mathbf{h}}\left(\frac{\cos^2\phi_{\mathbf{h}}}{\alpha_x^2}+\frac{\sin^2\phi_{\mathbf{h}}}{\alpha_y^2}\right)}$$

- Ward presents an approximation that is cheaper to compute, but Schlick found the direct vector implementation to be both exact and faster still:

$$f(\omega_i, \omega_o) = \frac{\mathbf{k}_d}{\pi} + \frac{\mathbf{k}_s}{4\pi\alpha_x\alpha_y\sqrt{(\mathbf{n}\cdot\omega_i)(\mathbf{n}\cdot\omega_o)}} e^{-\frac{\left(\frac{\mathbf{h}\cdot\mathbf{t}}{\alpha_x}\right)^2+\left(\frac{\mathbf{h}\cdot\mathbf{b}}{\alpha_y}\right)^2}{(\mathbf{h}\cdot\mathbf{n})^2}}$$

- Note: Because a squared dot-product of $\mathbf{h}$ appears in the numerator and denominator, we don't need to normalize $\mathbf{h}$

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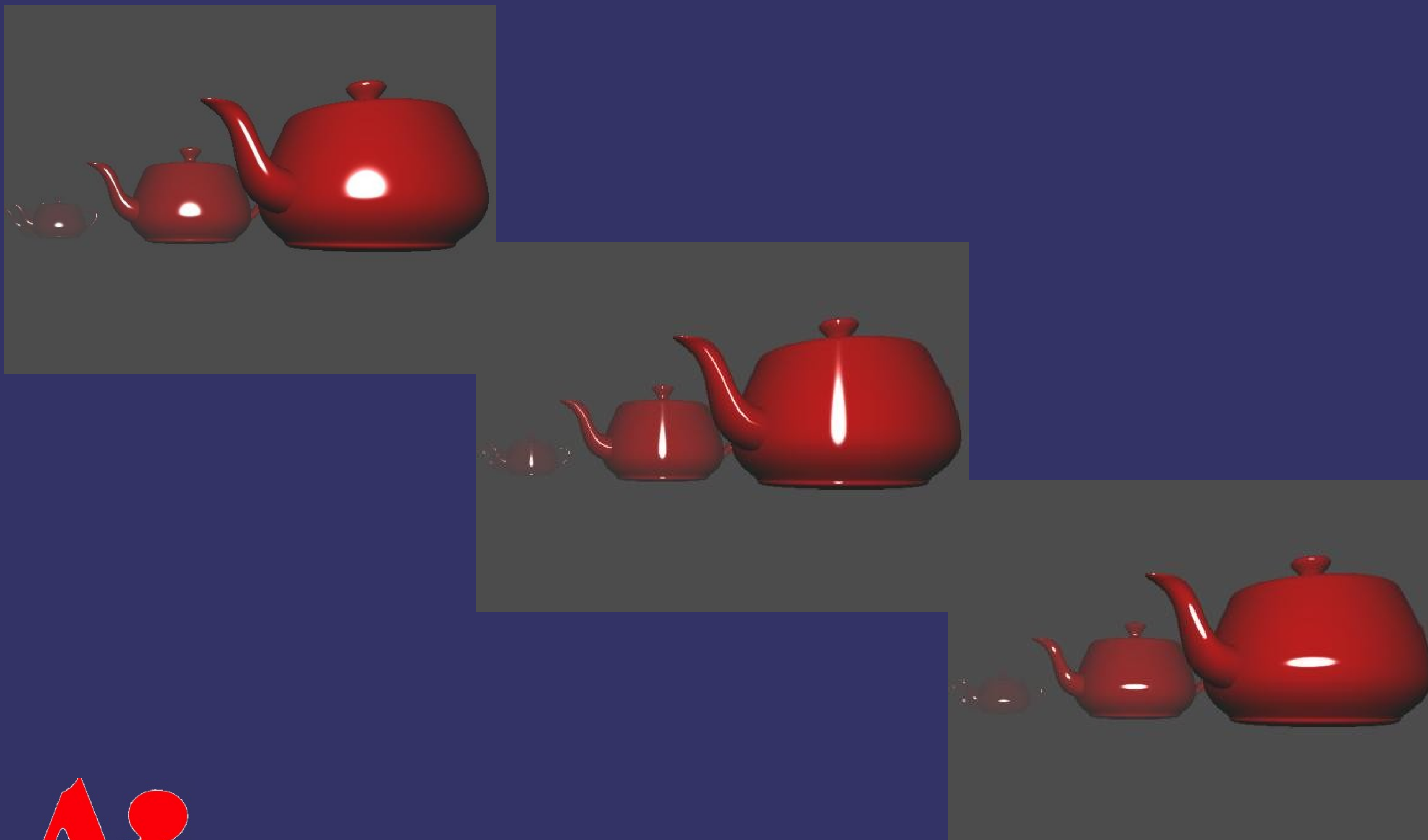
Ward's Anisotropic Model



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Ashikhmin Model

$$f_s(\omega_i, \omega_o) = \frac{\sqrt{(n_x+1)(n_y+1)}}{8\pi} \frac{(\hat{\mathbf{n}} \cdot \mathbf{h})^{n_x \cos^2 \phi_{\mathbf{h}} + n_y \sin^2 \phi_{\mathbf{h}}}}{(\mathbf{h} \cdot \omega) \max((\hat{\mathbf{n}} \cdot \omega_i), (\hat{\mathbf{n}} \cdot \omega_o))} F(\omega \cdot \mathbf{h})$$

- Most of the notation is the same as on the previous slides
  - This differs from the notation in Ashikhmin's paper
- n_x and n_y are Phong-like exponents that control the shape of the specular lobe
  - Roughly analogous to α_x and α_y in Ward's model
- $F(\theta)$ is the Fresnel term

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Ashikhmin Model

$$f_d(\omega_i,\omega_o)=\frac{28\,K_d}{23\,\pi}(1-F(0°))\left(1-\left(1-\frac{\mathbf{n}\cdot\omega_i}{2}\right)^5\right)\left(1-\left(1-\frac{\mathbf{n}\cdot\omega_o}{2}\right)^5\right)$$

- The strange constant factor is “designed to ensure energy conservation.”

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Anisotropic Reflections

- Both models allow control of specular highlight relative to some coordinate space
  - What coordinate space?
  - How can the orientation of the highlight be controlled?

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Anisotropic Reflections

- Both models allow control of specular highlight relative to some coordinate space
  - What coordinate space?
  - How can the orientation of the highlight be controlled?
- We're in surface space!
  - Rotate **t** and **b** by an angle
    - This is **t** and **b** in tangent space
  - Or use a “tangent map”
  - Or use procedural tangents
  - Or...

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Neil Blevins, “Anisotropic Reflections.” June 19th, 2002. http://www.neilblevins.com/cg_education/aniso_ref/aniso_ref.htm

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Walter, B. Notes on the Ward BRDF. *Technical report PCG-05-06*, Program of Computer Graphics, Cornell University, April 2005. http://www.graphics.cornell.edu/pubs/2005/Wal05.html

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E.M. Waves in Conductors

- Electromagnetic waves in conductors cause free electrons in the material to oscilate
  - The frequency of this oscillation is proportional to the frequency of the electromagnetic wave
  - These oscillations create eddy currents inside the material
  - These eddy currents force the primary current very near the surface
  - The change in current density w.r.t. change of depth is known as the *skin effect*

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E.M. Waves in Conductors

- Higher frequency waves cause the current to be limited to thinner and thinner skins on the material
  - A 1GHz wave in copper is restricted to ~0.5mm
  - A 60Hz wave in copper is restricted to ~10mm
  - Note: I'm trading a lot of physics here for a lot of hand waving!

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E.M. Waves in Conductors

- What does this have to do with lighting?!?

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E.M. Waves in Conductors

- What does this have to do with lighting?!?
  - Light is “just” an electromagnetic wave
  - Visible light is ~400THz - ~700THz
    - THz is tera-Hz or 1,000GHz
- As a result, light cannot penetrate deeply into metals
  - Diffuse reflection in dielectrics is primarily caused subsurface scattering
  - Lacking this, metal doesn't have a traditional diffuse reflection component
    - Cook & Torrance pointed this out as well

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Metals

- Two main components to metallic reflection:
  - A mostly pure specular component
    - A la Phong or Blinn
  - A *directional* diffuse component
    - Diffuse in the sense that the reflected color is the color of the material
- None of our current models have a directional diffuse component

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Lafortune BRDF

- Remember Phong:

$$\mathbf{k}=\mathbf{k}_s(\mathbf{v}\cdot\mathbf{r})^s\mathbf{i}_s$$

  - $\mathbf{r}$ is the ideal reflection vector
  - Calculation using vectors:

$$\mathbf{r}=2(\mathbf{n}\cdot\mathbf{l})\mathbf{n}-\mathbf{l}$$

  - Calculation using the Householder matrix:

$$\mathbf{r}=\mathbf{l}^T(2\mathbf{n}\mathbf{n}^T-\mathbf{I})=\mathbf{l}^T\mathbf{M}$$

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Lafortune BRDF

- If we're in surface space, what is $\mathbf{M}$?

$$\mathbf{r} = \mathbf{l}^{\mathrm{T}}\left(2\mathbf{n}\mathbf{n}^{\mathrm{T}} - \mathbf{I}\right) = \mathbf{l}^{\mathrm{T}}\mathbf{M}$$

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Lafortune BRDF

- If we're in surface space, what is $\mathbf{M}$?

$$\mathbf{r} = \mathbf{l}^T (2\mathbf{n}\mathbf{n}^T - \mathbf{I}) = \mathbf{l}^T \mathbf{M}$$

  - Remember: $\mathbf{n}$ = { 0, 0, 1 }

$$\begin{aligned}
\mathbf{M} &= 2\mathbf{n}\mathbf{n}^T - \mathbf{I} \\
&= 2\{0,0,1\}\{0,0,1\}^T - \mathbf{I} \\
&= 2\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \mathbf{I} \\
&= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

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Lafortune BRDF

- What if we could replace $\mathbf{M}$ with some other matrix?

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Lafortune BRDF

- What if we could replace $\mathbf{M}$ with some other matrix?
  - We could move the specular lobe!
  - The new matrix must be symmetric ($\mathbf{M} = \mathbf{M}^T$) or it will violate Helmoltz Reciprocity
  - It turns out that almost all cases except very unusual types of anisotropy, $\mathbf{M}$ is also diagonal
    - $\mathbf{c}_x = \mathbf{c}_y$ is also typical

$$\mathbf{M} = \begin{bmatrix} \mathbf{c}_x & 0 & 0 \\ 0 & \mathbf{c}_y & 0 \\ 0 & 0 & \mathbf{c}_z \end{bmatrix}$$

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Lafortune BRDF

- We can rearrange the math a bit:

$$
\begin{aligned}
\mathbf{k} &= \mathbf{k}_s \left(\mathbf{r}\cdot\mathbf{v}\right)^s \mathbf{i}_s \\
&= \mathbf{k}_s \left(\left(\mathbf{l}^T\mathbf{M}\right)\cdot\mathbf{v}\right)^s \mathbf{i}_s && \mathrm{r}=\mathbf{l}^T\mathbf{M} \\
&= \mathbf{k}_s \left(\left(\mathbf{M}^T\mathbf{l}\right)\cdot\mathbf{v}\right)^s \mathbf{i}_s \\
&= \mathbf{k}_s \left(\left(\mathbf{M}\mathbf{l}\right)\cdot\mathbf{v}\right)^s \mathbf{i}_s && \mathbf{M}=\mathbf{M}^T \\
&= \mathbf{k}_s \left(\left(\mathbf{c}*\mathbf{l}\right)\cdot\mathbf{v}\right)^s \mathbf{i}_s \\
&= \mathbf{k}_s \left(\mathbf{c}_x\mathbf{l}_x\mathbf{v}_x+\mathbf{c}_y\mathbf{l}_y\mathbf{v}_y+\mathbf{c}_z\mathbf{l}_z\mathbf{v}_z\right)^s \mathbf{i}_s
\end{aligned}
$$

- What if we could fit measured data to a series of cosine lobes?

$$\mathbf{k}=\sum_i \mathbf{k}_s\left(\mathbf{c}_{x,i}\mathbf{l}_x\mathbf{v}_x+\mathbf{c}_{y,i}\mathbf{l}_y\mathbf{v}_y+\mathbf{c}_{z,i}\mathbf{l}_z\mathbf{v}_z\right)^{s_i}\mathbf{i}_s$$

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Lafortune BRDF

- What does the data look like?
  - For matte steel:

	C_{xy}	C_z	s
Lobe 1, red	-1.11954	1.01272	15.8708
Lobe 1, green	-1.11845	1.01469	15.6489
Lobe 1, blue	-1.11999	1.01942	15.4571
Lobe 2, red	-1.05334	0.69541	111.267
Lobe 2, green	-1.06409	0.662178	88.9222
Lobe 2, blue	-1.08378	0.626672	65.2179
Lobe 3, red	-1.01684	1.00132	180.181
Lobe 3, green	-1.01635	1.00112	184.152
Lobe 3, blue	-1.01529	1.00108	195.773

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http://en.wikipedia.org/wiki/Skin_depth

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Next week...

- Fur and hair
  - Two final BRDFs
    - Grand unifying theory of anisotropic BRDFs
    - BRDFs for hair
  - Fins and shells

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