VGP352 – Week 4

- Agenda:
  - BRDFs, part 1
    - Common ideas and terminology
    - Cook-Torrance BRDF
    - Micro-facet based BRDFs
  - Hand in assignment #1
  - Start assignment #2

BRDF

- Bi-directional reflectance distribution function
  - Notation is $f(\omega_o, \omega_i)$

“…describes the ratio of reflected radiance exiting from a surface in a particular direction (defined by the vector ω_o) to the irradiance incident on the surface from direction ω_i over a particular waveband.”

BRDF

- In English...
  - Given an arbitrary input direction, ω_i, and an arbitrary output direction, ω_o, we can calculate the ratio of energy (light) transferred from ω_i to ω_o
- What does this tell us?

BRDF

- In English...
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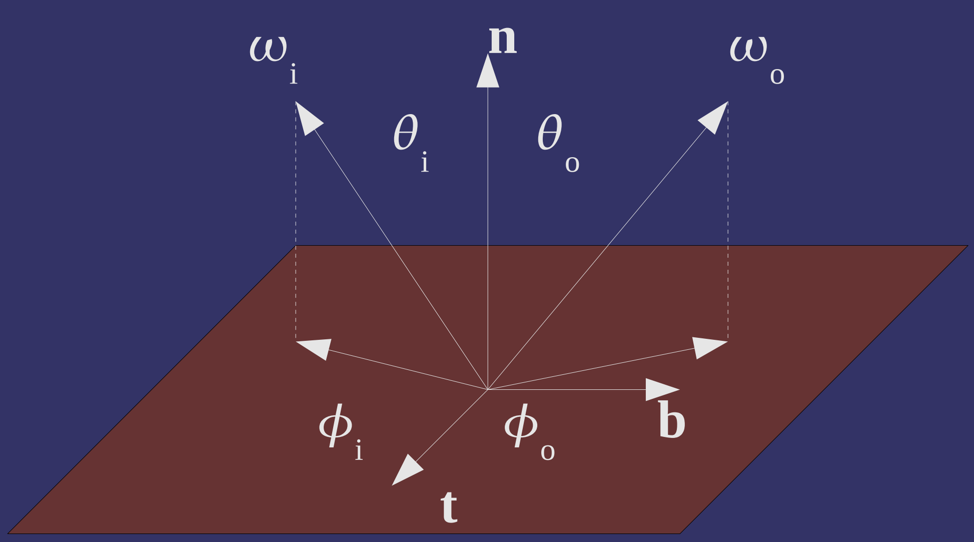
- What does this tell us?
  - If we know where the light is coming from, we can calculate how much of the light is reflected in any direction
  - If we know a light reflection direction (i.e., viewing direction) we can calculate the contribution of every possible light input direction

BRDF

- ω consists of the two angles:
  - θ is the elevation angle, and it is measured relative to the surface normal
  - ϕ is the azimuth angle, and it is measured relative to the surface tangent



BRDFs for Lighting

- ω is a *solid angle*

  "The solid angle, Ω, is the angle in three-dimensional space that an object subtends at a point. It is a measure of how big that object appears to an observer looking from that point."[1]

  - Each ω is a direction and a "slice" from the volume of the hemisphere around the point in question

[1] From http://en.wikipedia.org/wiki/Solid_angle

BRDFs for Lighting

- Why is it significant that ω is a solid angle?

BRDFs for Lighting

- Why is it significant that ω is a solid angle?
  - The size of a light from the POV of the receiver is significant
    - The ISS doesn't receive much illumination from Gliese 581, but it would if it were orbiting of one of Gliese 581's planets[1]
    - At 20.3 light years away, Gliese 581 has a tiny solid angle
    - At only ~2 million miles away, its solid angle is much larger
    - 0.00000024° vs. 1.28°
  - Could also use the area of the light projected onto a sphere around the receiver
    - As will be seen later, these units would not be convenient

[1] http://en.wikipedia.org/wiki/Gliese_581

BRDFs for Lighting

- The amount of light reflected from a particular input vector to a particular output vector:

$$L(\omega_o) = f(\omega_o, \omega_i) L(\omega_i) \cos\theta_i$$

Outgoing light intensity

A.k.a $\mathbf{n} \cdot \mathbf{l}$

Incoming light intensity

BRDFs for Lighting

- What if we want to calculate the amount light reflected to a particular output vector from *all possible* input vectors?

BRDFs for Lighting

- What if we want to calculate the amount light reflected to a particular output vector from *all possible* input vectors?

$$L(\omega_o) = \int_{\Omega} f(\omega_o, \omega_i) L(\omega_i) \cos\theta_i \, d\omega_i$$

  - Integration over a solid angle works just like any other integration
  - This integral is over the hemisphere above the point
    - This is a solid angle of 2π
  - Most BRDFs will contain a 1/π factor because of this

BRDF Properties

- Physically based BRDFs have two important properties:
  - Helmoltz reciprocity:

$$f(\omega_i, \omega_o) = f(\omega_o, \omega_i)$$

    - Also called Helmoltz Stereopsis
    - This is the “bi-directional” part of BRDF
  - Conservation of energy:

$$\forall \omega_i, \int_{\Omega} f(\omega_i, \omega_o) \cos\theta_o \, d\omega_o \leq 1$$

Where do BRDFs come from?

- Measured BRDFs
  - Measure every possible output from every possible output
  - Oregon BRDF Library (and others) have data captured from these instruments available

Measured BRDFs

Image from http://www.merl.com/projects/facescanning/

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Measured BRDFs

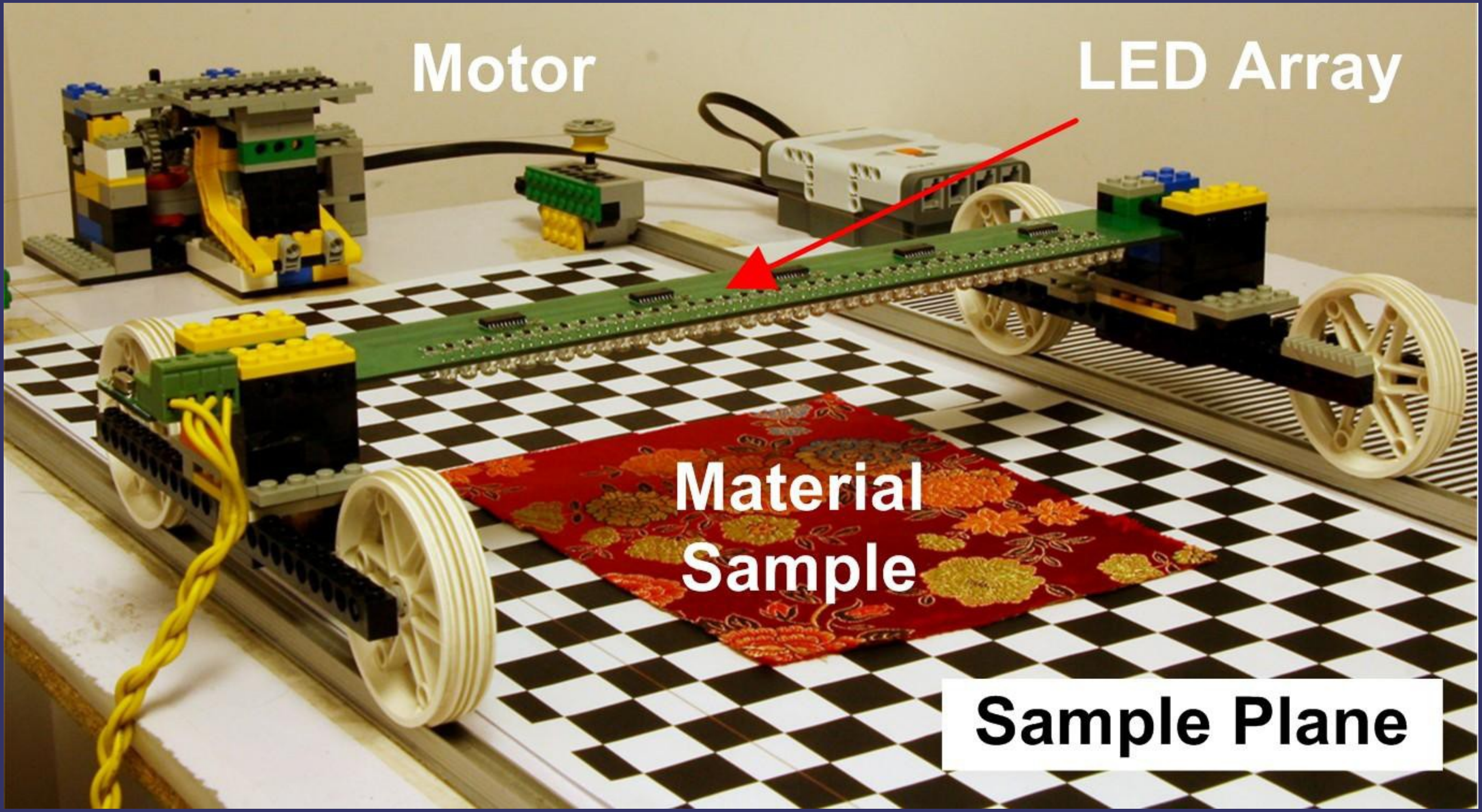

Image from http://www.shuangz.com/projects/aniso/

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References

Wang, J., Zhao, S., Tong, X., Snyder, J., and Guo, B. 2008. Modeling anisotropic surface reflectance with example-based microfacet synthesis. In *ACM SIGGRAPH 2008 Papers* (Los Angeles, California, August 11 - 15, 2008). SIGGRAPH '08. ACM, New York, NY, 1-9. http://www.shuangz.com/projects/aniso/

Sample BRDF data sets:

http://www.graphics.cornell.edu/online/measurements/reflectance/index.html

http://www1.cs.columbia.edu/CAVE//software/curet/

http://math.nist.gov/~FHunt/appearance/obl.html

Where do BRDFs come from?

- Measured BRDFs
  - Measure every possible output from every possible output
  - Oregon BRDF Library (and others) have data captured from these instruments available
- Analytical BRDFs
  - Mathematical models used to reproduce observed behavior
  - May be derived from simplified measured data

Cook-Torrance BRDF

- One of the oldest BRDFs used in graphics
  - Published by Robert Cook and Ken Torrance in 1982
    - Cook was at Lucasfilm, Ltd.
    - Torrance was at Cornell
  - Based on *micro-facets*

Micro-Facet Primer

- Surfaces are made of numerous infinitesimal subsurfaces that act as perfect mirrors
  - Distribution of the normals of these subsurfaces determines how specular the surface appears
  - Micro-facets can obscure other micro-facets both from the light and from the viewer
- We'll dive deep into both these aspects soon...

Cook-Torrance BRDF

$$
\begin{aligned}
f(\omega_o, \omega_i) &= \mathbf{k}_d f_d + \mathbf{k}_s f_s(\omega_o, \omega_i) \\
f_d &= 1/\pi \\
f_s(\omega_o, \omega_i) &= 1/\pi \frac{F \times D(\mathbf{n}\cdot\mathbf{h}) \times G(\mathbf{n}\cdot\omega_i, \mathbf{n}\cdot\mathbf{h}, \mathbf{n}\cdot\omega_o)}{(\mathbf{n}\cdot\omega_i)(\mathbf{n}\cdot\omega_o)}
\end{aligned}
$$

- F is the Fresnel factor
- D is the distribution of micro-facet normals
- G is the geometry occlusion factor
- $\mathbf{h}$ is the half-vector from the Blinn-Phong lighting equation

Micro-facet Distribution

- Micro-facet normals are random, but follow some distribution function
  - Sometimes call the *normal distribution function* (NDF)
  - Several models exist
  - Cook-Torrance uses the Beckmann Distribution:

$$D(\mathbf{n}\cdot\mathbf{h}) = \frac{1}{4\,m^2(\mathbf{n}\cdot\mathbf{h})^4}\,e^{-\left(\frac{1-(\mathbf{n}\cdot\mathbf{h})^2}{(\mathbf{n}\cdot\mathbf{h})^2 m^2}\right)}$$

  - m is a parameter that controls the smoothness of the surface

Geometry Occlusion Factor

- Represents the decrease in light transmission caused by occlusion of the light or viewer by other micro-facets

$$G(\mathbf{n}\cdot\omega_i, \mathbf{n}\cdot\mathbf{h}, \mathbf{n}\cdot\omega_o) = \min\left(1, \frac{2(\mathbf{n}\cdot\mathbf{h})(\mathbf{n}\cdot\omega_o)}{\omega\cdot\mathbf{h}}, \frac{2(\mathbf{n}\cdot\mathbf{h})(\mathbf{n}\cdot\omega_i)}{\omega\cdot\mathbf{h}}\right)$$

- Why aren't there any subscripts on ω in the denominators?
  - Hint: $\omega_i \cong \mathbf{l}$ and $\omega_o \cong \mathbf{v}$

Geometry Occlusion Factor

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$$G(\mathbf{n}\cdot\omega_i, \mathbf{n}\cdot\mathbf{h}, \mathbf{n}\cdot\omega_o) = \min\left(1, \frac{2(\mathbf{n}\cdot\mathbf{h})(\mathbf{n}\cdot\omega_o)}{\omega\cdot\mathbf{h}}, \frac{2(\mathbf{n}\cdot\mathbf{h})(\mathbf{n}\cdot\omega_i)}{\omega\cdot\mathbf{h}}\right)$$

- Why aren't there any subscripts on ω in the denominators?
  - Hint: $\omega_i \cong \mathbf{l}$ and $\omega_o \cong \mathbf{v}$
  - $\mathbf{h}$ is half way between $\mathbf{v}$ and $\mathbf{l}$:

$$\angle\,\mathbf{l}\,\mathbf{h} = \angle\,\mathbf{v}\,\mathbf{h} \therefore (\mathbf{h}\cdot\omega_i) = (\mathbf{h}\cdot\omega_o)$$

Cook-Torrance Diffuse Factor

- Cook-Torrance diffuse factor:

$$f_d = 1/\pi$$

- "Typical" diffuse factor:

$$\mathbf{k}_d = \mathbf{n} \cdot \mathbf{l}$$

- Remember how the BRDF is used:

$$L(\omega_o) = f(\omega_o, \omega_i) L(\omega_i) \cos\theta_i$$

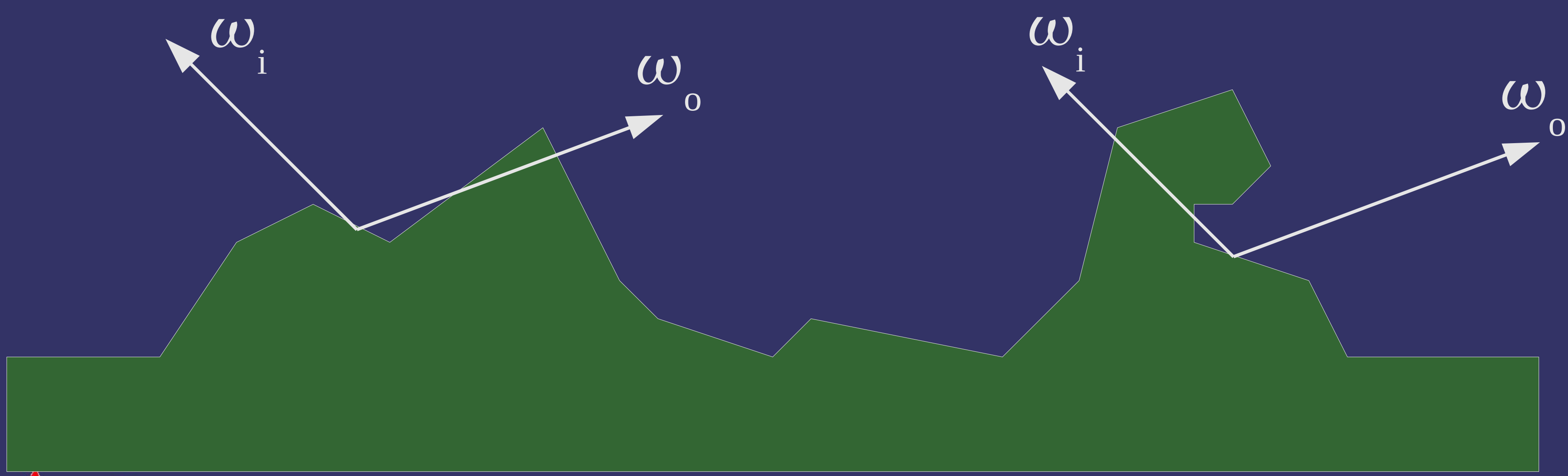
  - We just want to scale the incoming energy by the total angle and let the built in $(\mathbf{n} \cdot \omega_i)$ do the rest
  - Remember $\omega_i \cong \mathbf{l}$

Micro-Facet Deep Dive

- Surfaces are made of numerous infinitesimal subsurfaces that act as perfect mirrors
  - Each facet only reflects light along the ideal reflection vector
  - Determining the number of visible facets for a given $\mathbf{v}$ and $\mathbf{l}$ is enough to determine the BRDF

Micro-Facet Deep Dive

- Add two assumptions:
  - Facet normals are distributed randomly according to some distribution function $p(\mathbf{h})$
  - A facet only contributes if it is visible to both $\mathbf{v}$ and $\mathbf{l}$



Micro-Facet Deep Dive

- BRDF is determined by:
  - Fresnel term
  - Fraction of micro-facets with $\mathbf{n} = \mathbf{h}$
  - Fraction of micro-facets visible to both $\mathbf{l}$ and $\mathbf{v}$
    - Non-visible to $\mathbf{l}$ is often called “shadowing”
    - Non-visible to $\mathbf{v}$ is often called “masking”
    - Both can just be called “occlusion”

Normal Distribution

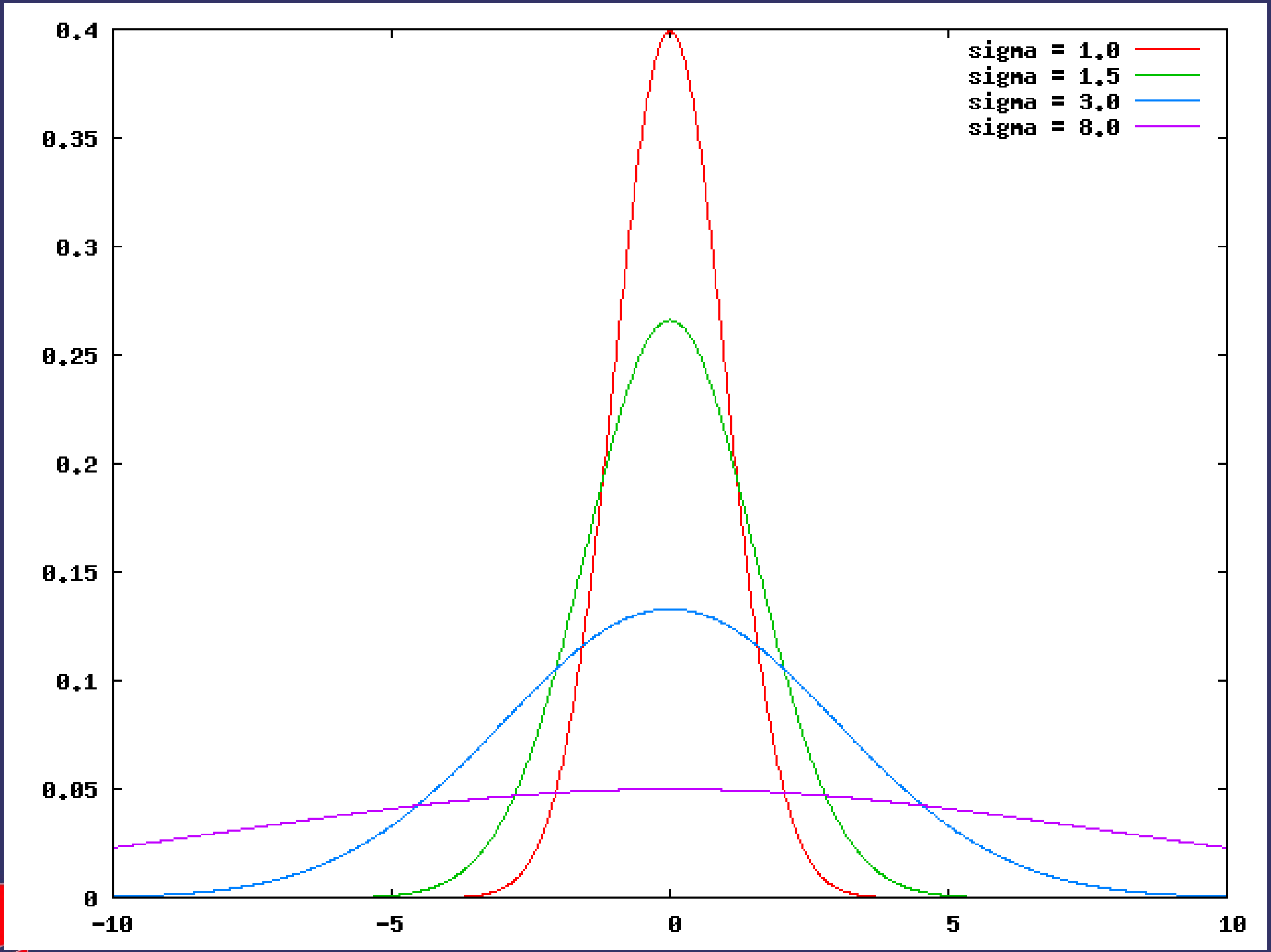
- Given **n**, determine the fraction of micro-facet normals that point towards **h**
  - Can use arbitrary function to calculate this probability
    - May be convenient to encode this in a texture
  - Gaussian or *standard normal distribution* function seems like a good choice
    - The more different the **h** is from **n**, the lower the probability

Gaussian Distribution

$$P(\theta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{\theta^2}{2\sigma^2}\right)}$$

σ is the standard deviation

Gaussian Distribution



Gaussian Distribution

$$P(\theta)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\left(\frac{\theta^2}{2\sigma^2}\right)}$$

σ is the standard deviation

- Looking at the graph, why is this distribution unsuitable?

Gaussian Distribution

$$P(\theta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{\theta^2}{2\sigma^2}\right)}$$

σ is the standard deviation

- Looking at the graph, why is this distribution unsuitable?
  - As σ increases, the effective range increases to ∞
  - Distribution is based on θ, but we only know $\cos(\theta)$

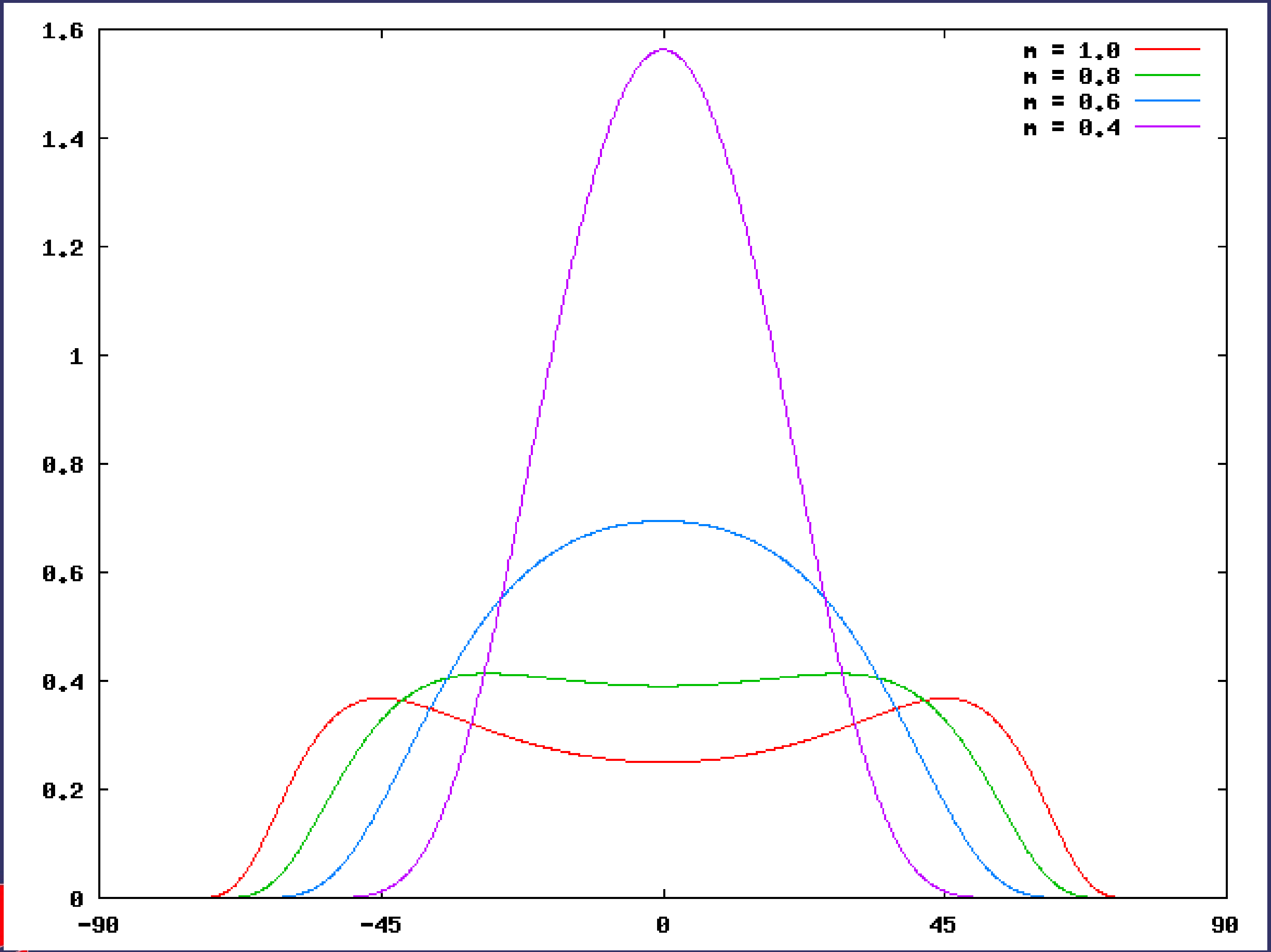
Beckmann Distribution

$$P(\theta) = \frac{1}{4m^2\cos^4\theta} e^{-\left(\frac{\tan^2\theta}{m^2}\right)}$$

m is average slope of the surface micro-facets

- Physically based model of rough surfaces
  - Based on Beckmann's research in the early 60s
- All calculations are based on $\cos(\theta)$!
  - Remember: $\tan^2(\theta)$ is (1 - $\cos^2(\theta)$) / $\cos^2(\theta)$

Beckmann Distribution



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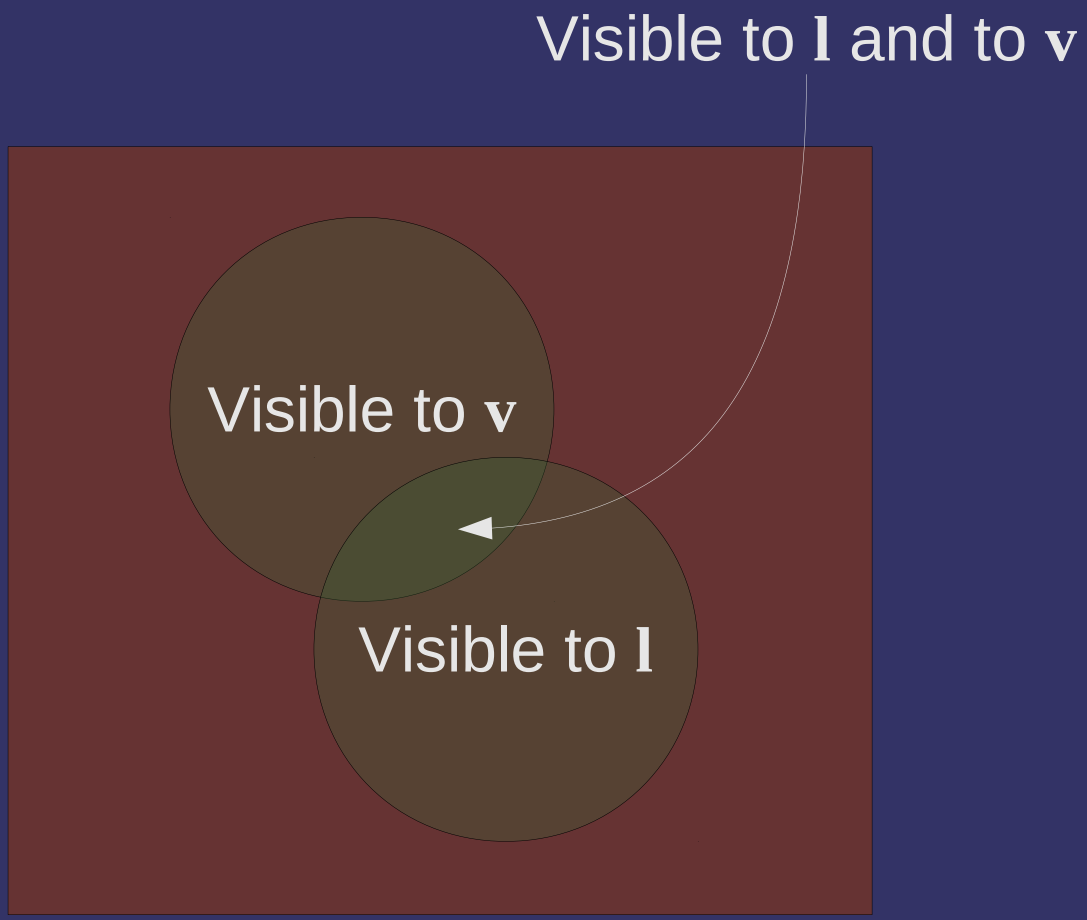
Micro-facet Occlusion

- Determine the probability of a facet being visible to the light and to the viewer
  - Use one probability function, $P_{vis}(\theta)$, for the probability of visibility to either $\mathbf{l}$ or $\mathbf{v}$
  - Assume that visibility and orientation are uncorrelated

Micro-facet Occlusion

- Given $P_{vis}(\theta_{\mathbf{v}})$ and $P_{vis}(\theta_{\mathbf{l}})$, what is $P_{vis}(\theta_{\mathbf{v}} \cap \theta_{\mathbf{l}})$?

Micro-facet Occlusion

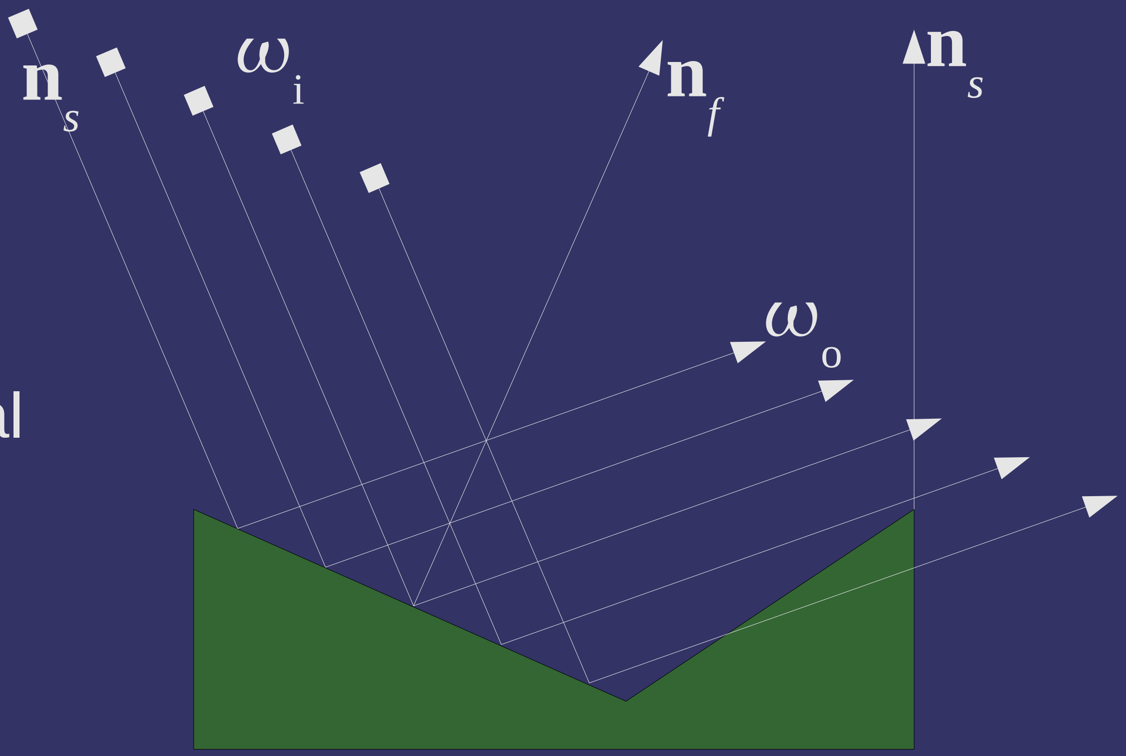


Micro-facet Occlusion

- Given $P_{vis}(\theta_v)$ and $P_{vis}(\theta_l)$, what is $P_{vis}(\theta_v \cap \theta_l)$?
  - Generating a new probability function from dependent probability functions is a difficult problem in general
  - $P_{vis}(\theta_v) \times P_{vis}(\theta_l) < P_{vis}(\theta_v \cap \theta_l)$
    - $P(A)P(B) = P(A \cap B) \leftrightarrow A$ and B are independent
    - Visibility to the light and viewer are not independent
      - Example: Put the light and viewer at the same location
  - Cook and Torrance suggest $\min(P_{vis}(\theta_v), P_{vis}(\theta_l))$
  - Other methods exist... the reading for next week contains one

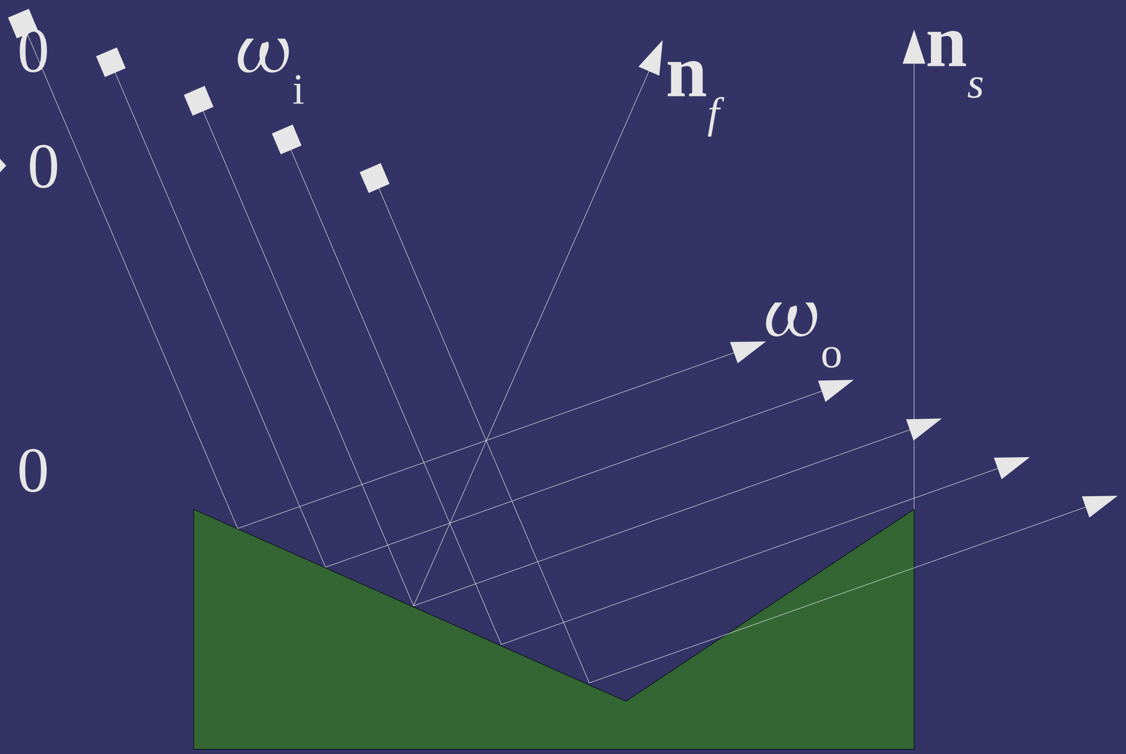
Micro-facet Occlusion

- How do we estimate $P_{vis}(\theta)$?
  - Clearly ω_i, ω_o, $\mathbf{n}_f$, and $\mathbf{n}_s$ are involved
    - $\mathbf{n}_f$ is the facet normal
    - $\mathbf{n}_s$ is the surface normal



Micro-facet Occlusion

- Observations:
  - Occlusion increases as:
    - $\angle \mathbf{n}_f \mathbf{n}_s \rightarrow 90^\circ \Leftrightarrow (\mathbf{n}_f \cdot \mathbf{n}_s) \rightarrow 0$
    - $\angle \omega \mathbf{n}_s \rightarrow 90^\circ \Leftrightarrow (\omega \cdot \mathbf{n}_s) \rightarrow 0$
  - Occlusion decreases as:
    - $\angle \omega \mathbf{n}_f \rightarrow 90^\circ \Leftrightarrow (\omega \cdot \mathbf{n}_f) \rightarrow 0$

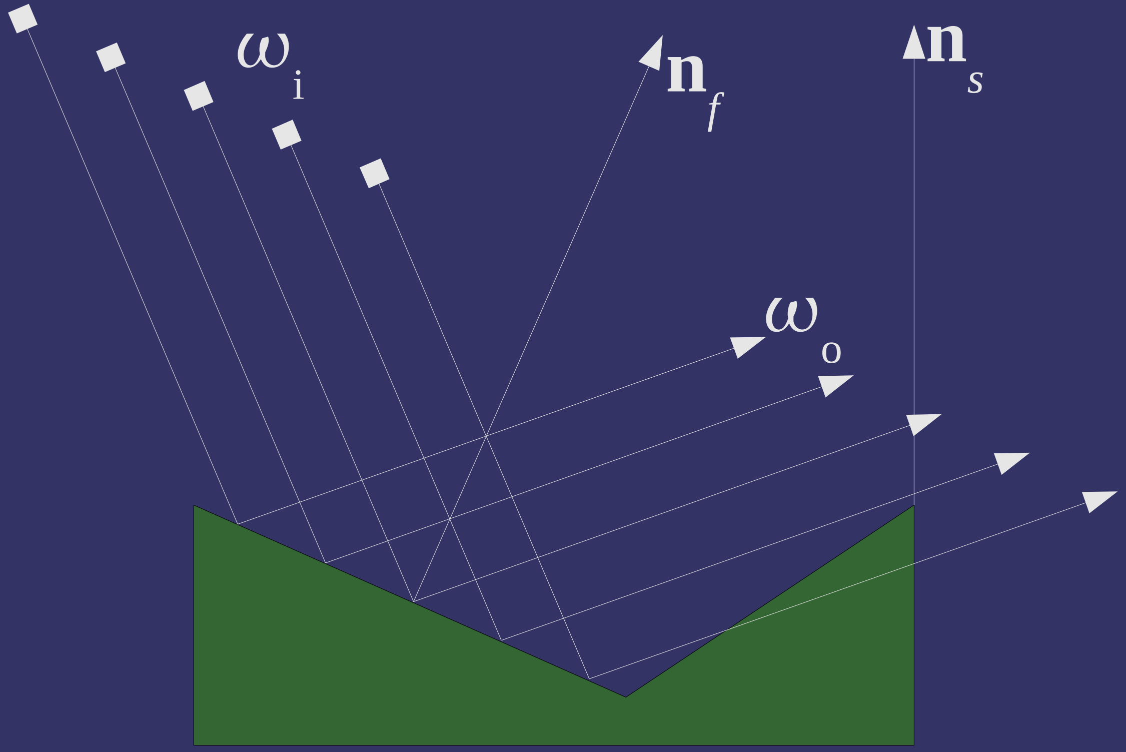


Micro-facet Occlusion

- Cook-Torrance uses:

$$P_v(\theta) = \frac{2(\mathbf{n}_s \cdot \mathbf{n}_f)(\mathbf{n}_s \cdot \omega)}{\omega \cdot \mathbf{n}_f}$$

- What other vector is equivalent to $\mathbf{n}_f$?



Micro-facet Occlusion

- Cook-Torrance uses:

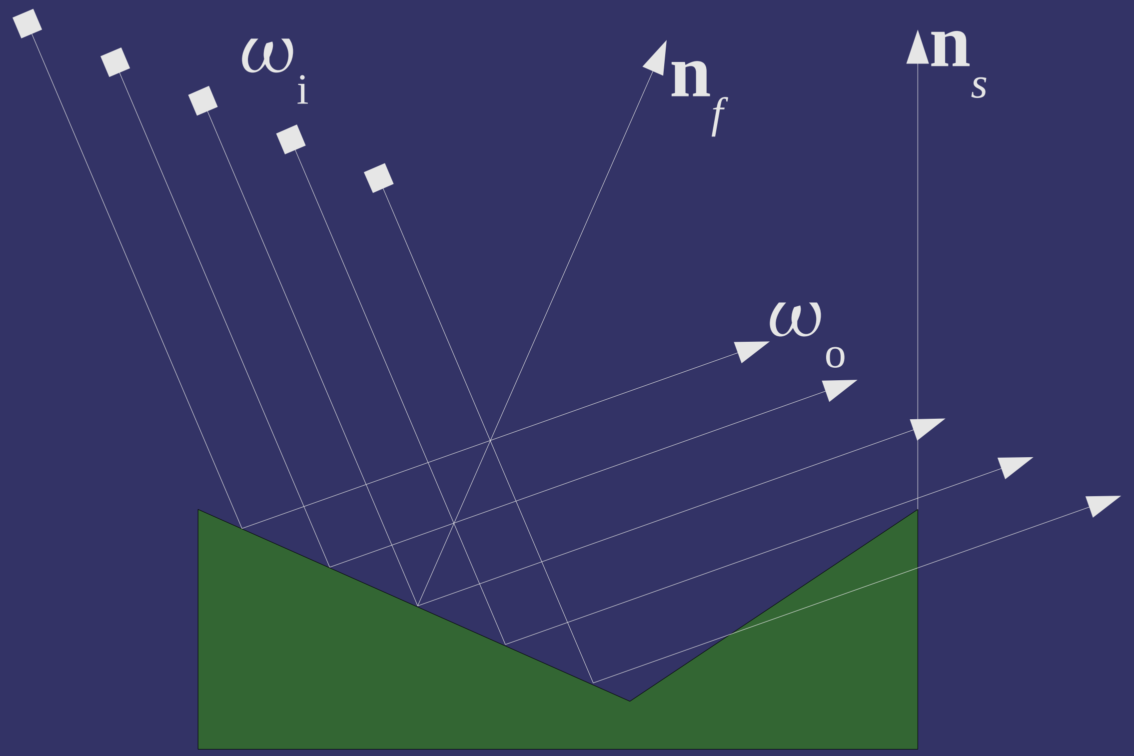
$$P_v(\theta) = \frac{2(\mathbf{n}_s \cdot \mathbf{n}_f)(\mathbf{n}_s \cdot \omega)}{\omega \cdot \mathbf{n}_f}$$

- What other vector is equivalent to $\mathbf{n}_f$?
  - By definition, $\mathbf{n}_f = \mathbf{h}$

$$G_v = \frac{2(\mathbf{n} \cdot \mathbf{h})(\mathbf{n} \cdot \mathbf{v})}{\mathbf{v} \cdot \mathbf{h}}$$

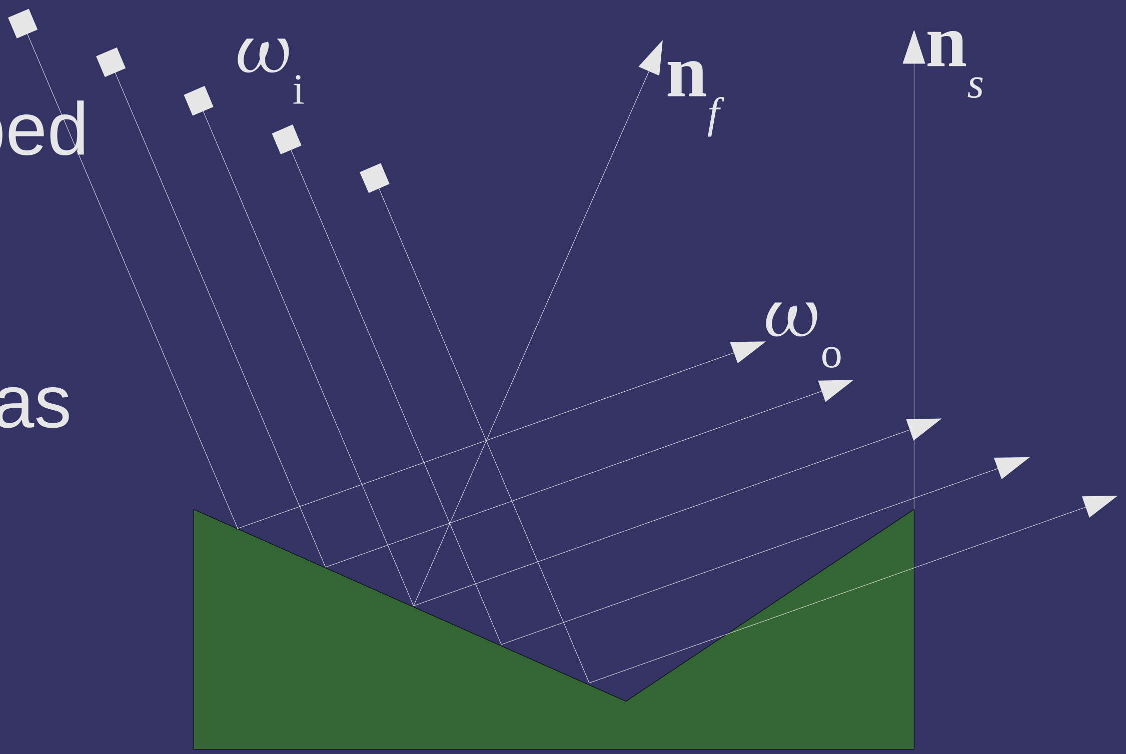
$$G_l = \frac{2(\mathbf{n} \cdot \mathbf{h})(\mathbf{n} \cdot \mathbf{l})}{\mathbf{l} \cdot \mathbf{h}}$$

$$\mathbf{l} \cdot \mathbf{h} = \mathbf{v} \cdot \mathbf{h}$$



Micro-facet Occlusion

- This turns out to be a poor model
  - Real surfaces aren't made of long, V-shaped channels
  - This reading for next week addresses this as well



References

http://wiki.gamedev.net/index.php/D3DBook:(Lighting)_Cook-Torrance

Philip Dutré. “Global Illumination Compendium.” Computer Graphics, Department of Computer Science Katholieke Universiteit Leuven. 2003. http://www.cs.kuleuven.ac.be/~phil/GI/

Reading for Next Week

Prepare for next week:

Ashikmin, Michael and Premože, Simon and Shirley, Peter, "A microfacet-based BRDF generator." In *SIGGRAPH ’00: Proceedings of the 27th Annual Conference on Computer Graphics and Interactive Techniques*, pages 65–74. ACM Press/Addison-Wesley Publishing Co., 2000. http://www.cs.utah.edu/~shirley/papers/facets.pdf

Next week...

- Quiz #2
- More BRDFs
  - Anisotropic reflection
    - Ward BRDF
    - Ashikhmin BRDF
  - Metals
    - How do metals “reflect” light?
    - Lafortune BRDF

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