VGP352 – Week 1

- Agenda:
  - Course Intro
  - Curves
  - Curved surfaces
  - Per-fragment lighting revisited
    - Phong Shading
    - Surface-space
  - Bump mapping
    - Basic usage
    - Bumpmap storage

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What should you already know?

- C++ and object oriented programming
  - For most assignments you will need to implement classes or portions of classes that conform to specific interfaces
- Graphics terminology and concepts
  - Polygon, pixel, texture, infinite light, point light, spot light, etc.
- Linear algebra and vector math
  - Matrix arithmetic

What should you already know?

- Material from VGP351:
  - Using OpenGL
    - Setting up shaders
    - Getting data in
    - etc.
  - Transformations
    - 3D space transformations
    - Projections
  - Lighting and shading
  - Texture mapping

What will you learn?

- Advanced lighting models
  - BRDFs
  - Fur and hair rendering
  - “Toon” and other non-photorealistic rendering

How will you be graded?

- Four bi-weekly quizzes
  - These are listed on the syllabus
- One final exam
- Three programming projects
  - The first will be pretty small...perhaps small enough to complete in class
  - The remaining two projects will be larger
- One in-class presentation

How will you be graded?

- Keep in mind:
  - There is a *lot more* reading than in VGP351
    - More readings from the textbook
    - Readings from academic papers
  - There is *more* programming than in VGP351

How will programs be graded?

- Does the program produce the correct output?
- Are appropriate algorithms and data-structures used?
- Is the code readable, clear, and properly documented?

How will the presentation be graded?

- During the term, several papers will be assigned to read
  - Select and present one of the assigned readings to the class
    - What is the problem being solved?
    - How does the paper's author solve that problem?
    - What is novel about the author's solution?
    - What questions does the paper leave unanswered?
  - Material from some papers may appear on bi-weekly quizzes

Class Web Site

- Syllabus, assignments, and base code:

http://people.freedesktop.org/~idr/2010Q1-VGP352/

Camera Control

- How can we move a virtual camera through a series of artist selected positions?

Camera Control

- How can we move a virtual camera through a series of artist selected positions?
  - Linearly interpolate between the positions

$$\begin{aligned}\mathbf{p}(t) &= \mathbf{p}_0 + t(\mathbf{p}_1 - \mathbf{p}_0)\\ &= (1-t)\mathbf{p}_0 + t\,\mathbf{p}_1\end{aligned}$$

  - Results in a function that is positionally continuous
    - Also known as C^0 continuity
- What's wrong with C^0?

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  - Results in a function that is positionally continuous
    - Also known as C^0 continuity
- What's wrong with C^0?
  - Jarring change in direction at control points
  - Jarring change in speed at control points
    - Direction change or speed change = velocity change

Camera Control

- How can we fix this?
  - Apply linear interpolation *again*
  - Also add an additional control point
    - Now have $\mathbf{p}_0$, $\mathbf{p}_1$, and $\mathbf{p}_2$
- To calculate $\mathbf{p}(t)$:
  - Lerp between $\mathbf{p}_0$ and $\mathbf{p}_1$, call the result $\mathbf{d}$
  - Lerp between $\mathbf{p}_1$ and $\mathbf{p}_2$, call the result $\mathbf{e}$
  - Lerp between $\mathbf{d}$ and $\mathbf{e}$
- Formally, this is a *Bézier curve*
  - Pronounced *beh-zee-eh*

Bézier Curve

- This works out to:

$$\mathbf{p}(t)=(1-t)^2\mathbf{p}_0+2t(1-t)\mathbf{p}_1+t^2\mathbf{p}_2$$

- More formally:

$$\mathbf{p}_i^k(t)=(1-t)\mathbf{p}_i^{k-1}(t)+t\,\mathbf{p}_{i+1}^{k-1}(t),\begin{cases}\mathrm{k} & = & 1..n\\ \mathrm{i} & = & 0..n-k\end{cases}$$

  - Curve with x control points is degree x-1
    - n is the degree of the polynomial that defines the curve
    - Our curve with 3 control points is degree 2
  - The initial control points are $\mathbf{p}_i^0$ but are written $\mathbf{p}_i$

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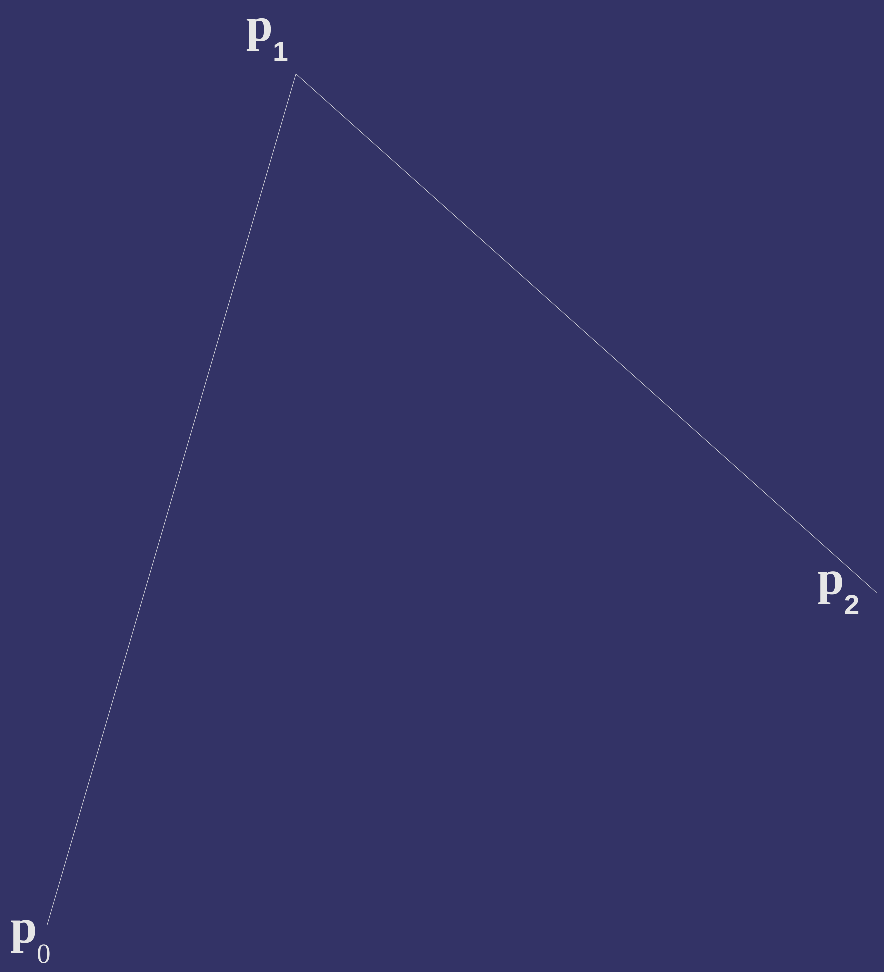
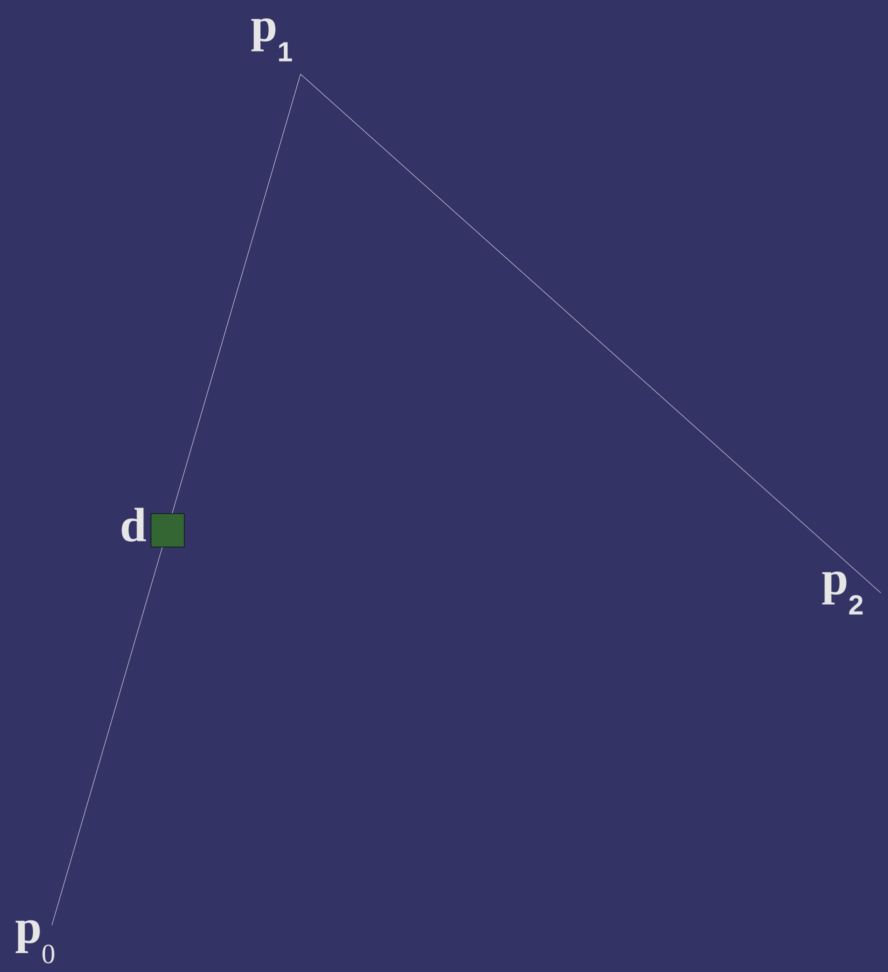


Bézier Curve



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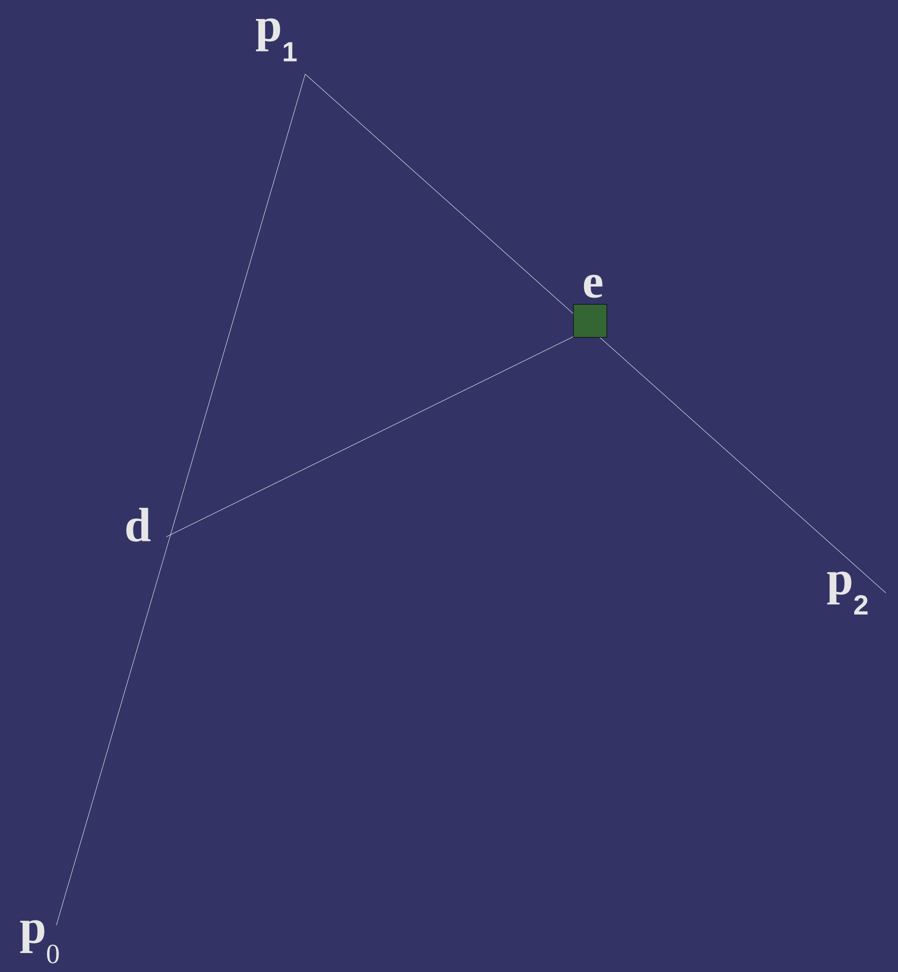

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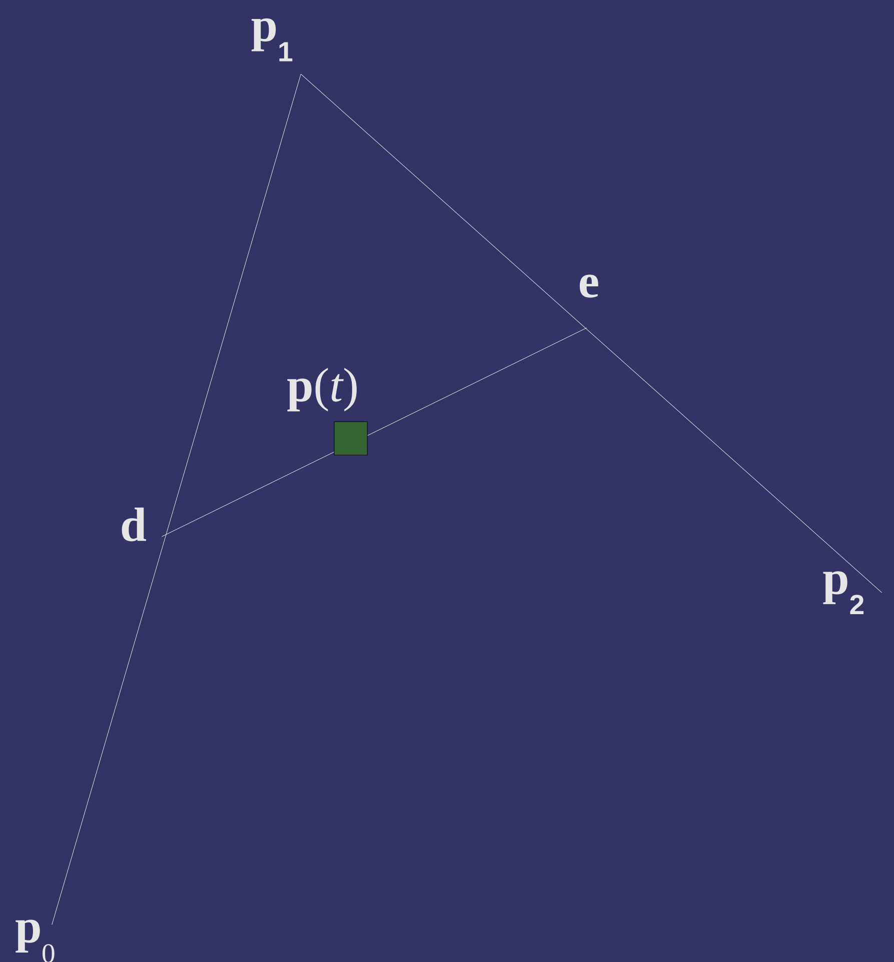
Bézier Curve



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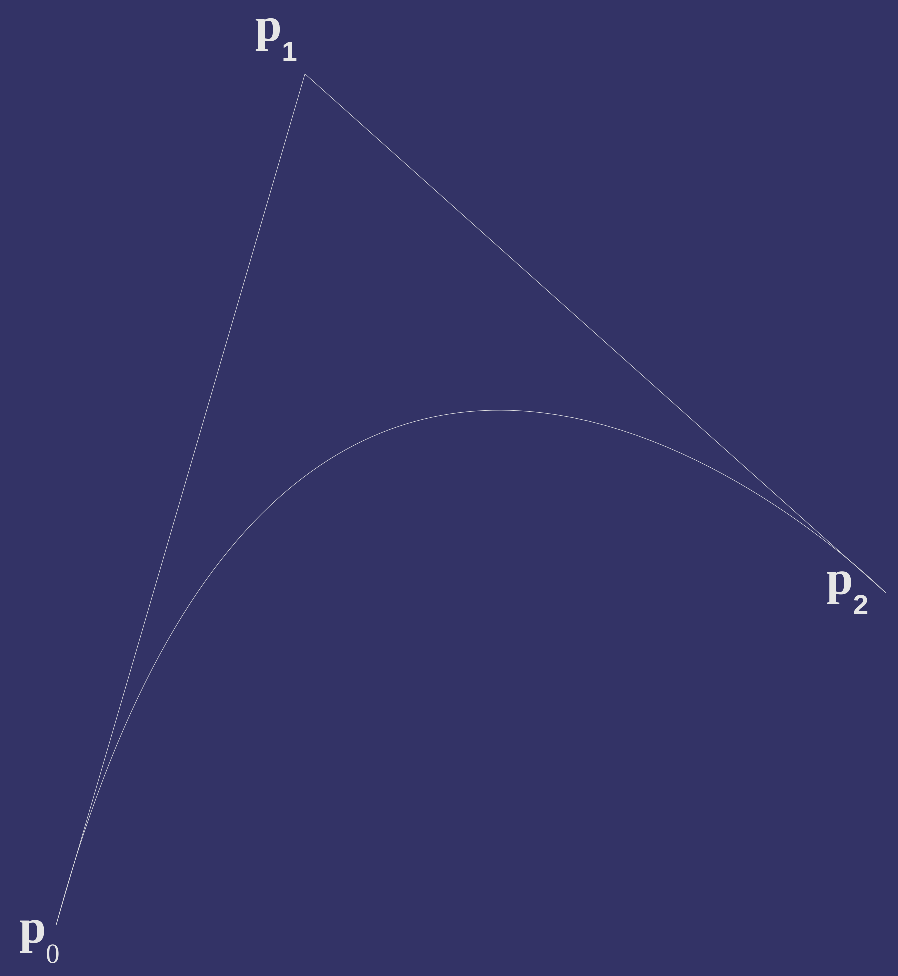
Bézier Curve



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Bézier Curve

- Note:
  - Curve lies within the convex hull of the control points
  - Curve only passes through $\mathbf{p}_0$ and $\mathbf{p}_n$

Bézier Curve

- Repeated interpolation is cumbersome
  - Also inefficient for large n
- Can we do better?

Bézier Curve

- Repeated interpolation is cumbersome
  - Also inefficient for large n
- Can we do better?
  - Yes!
  - We can use *algebra* instead of interpolation

Bézier Basis Functions

- Rewrite a weighted sum of control points:

$$\mathbf{p}(t) = \sum_{i=0}^{n} \mathrm{B}_i^n(t)\,\mathbf{p}_i$$

$$\mathrm{B}_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}$$

$$= \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}$$

  - B_i^n is the “Bernstein polynomial” or “Bézier basis function”
  - Note:

$$t \in [0,1] \rightarrow B_i^n(t) \in [0,1]$$

$$\sum_{i=0}^{n} B_i^n(t) = 1$$

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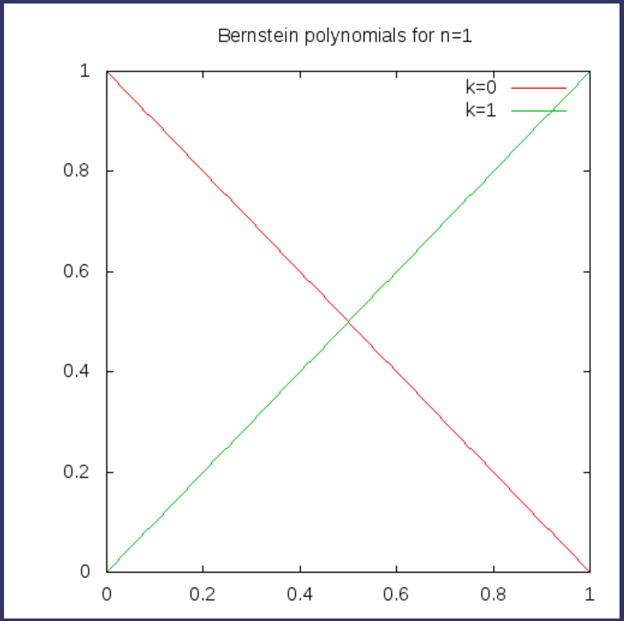
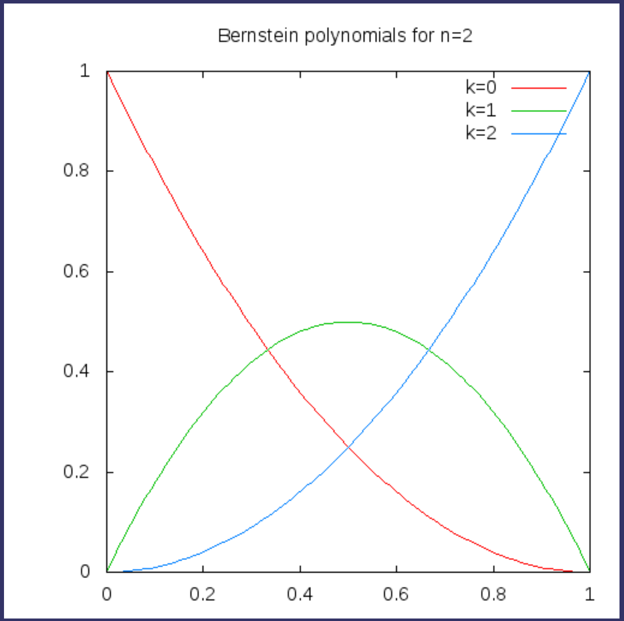
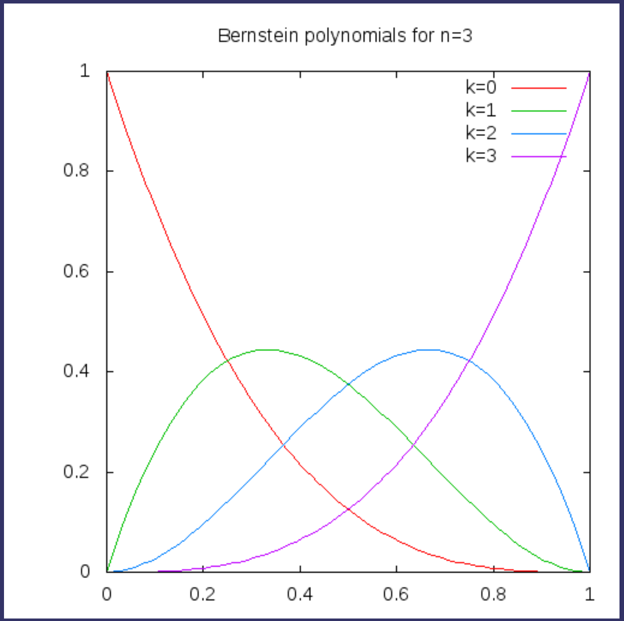
Bézier Basis Functions







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Bézier Curve

- Usually unnecessary to go higher than n=3
  - Why?

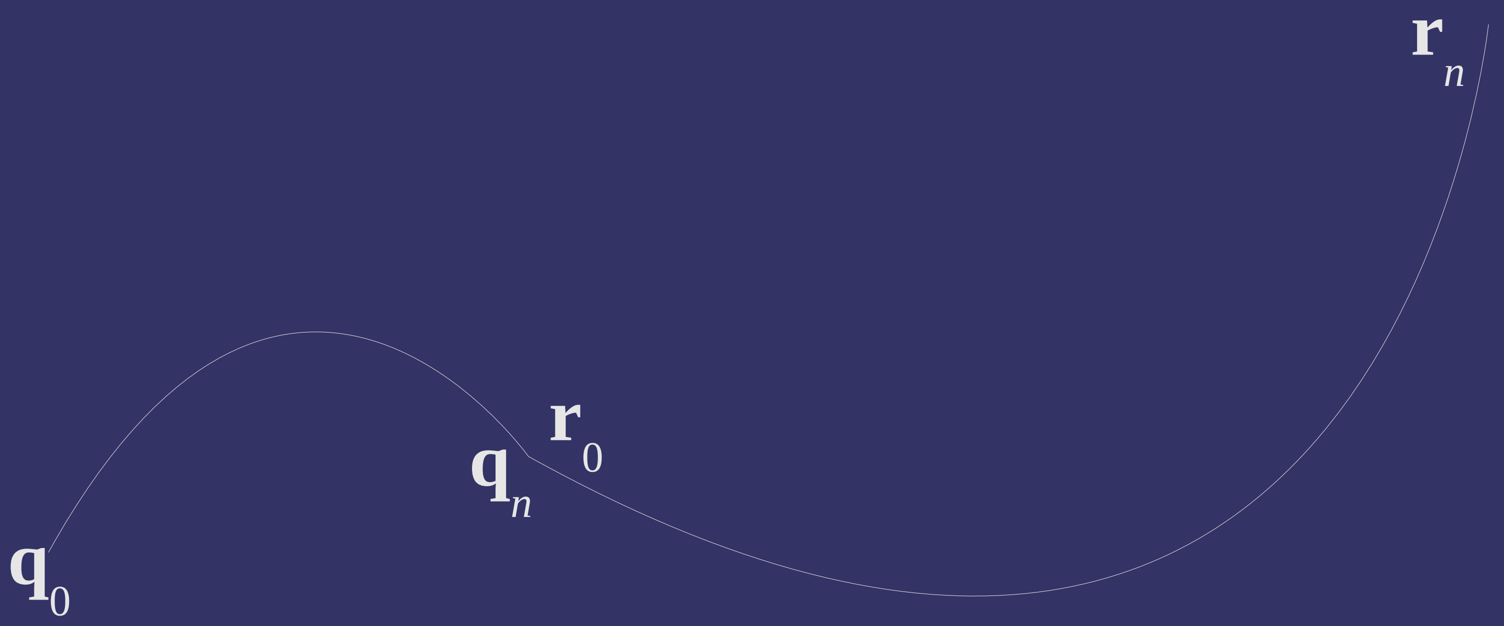
Bézier Curve

- Usually unnecessary to go higher than n=3
  - Why?
  - Evaluation cost increases as n increases
  - Cubic polynomials are the lowest degree whose derivative can change direction
    - This allows multiple cubic Bézier curves to be combine to approximate most shapes

Piecewise Bézier Curves

- Curve only passes through $\mathbf{p}_0$ and $\mathbf{p}_n$
  - For camera control, we need to hit other definable points
- Define multiple curves
  - Control points $\mathbf{q}_i$, $\mathbf{r}_i$, $\mathbf{s}_i$, etc.
  - Set $\mathbf{q}_n = \mathbf{r}_0$
    - This is called a *joint*

Piecewise Bézier Curves



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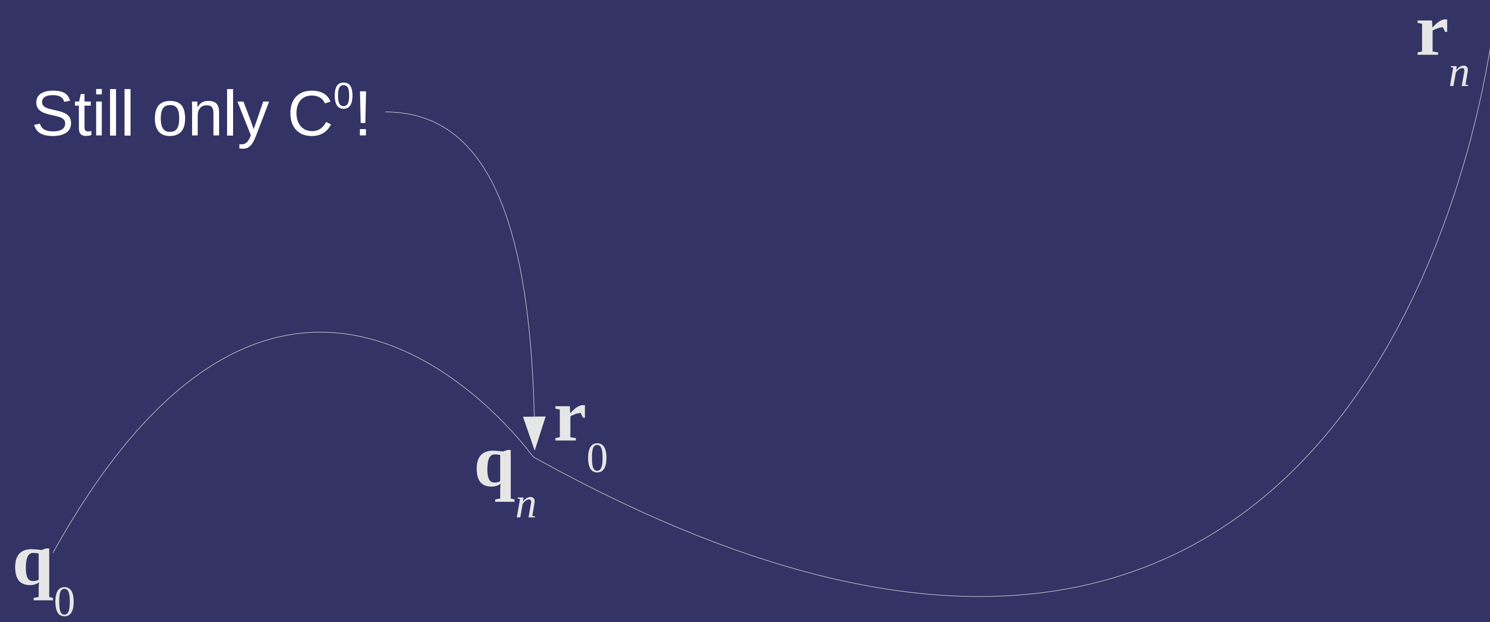
Piecewise Bézier Curves



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Piecewise Bézier Curves

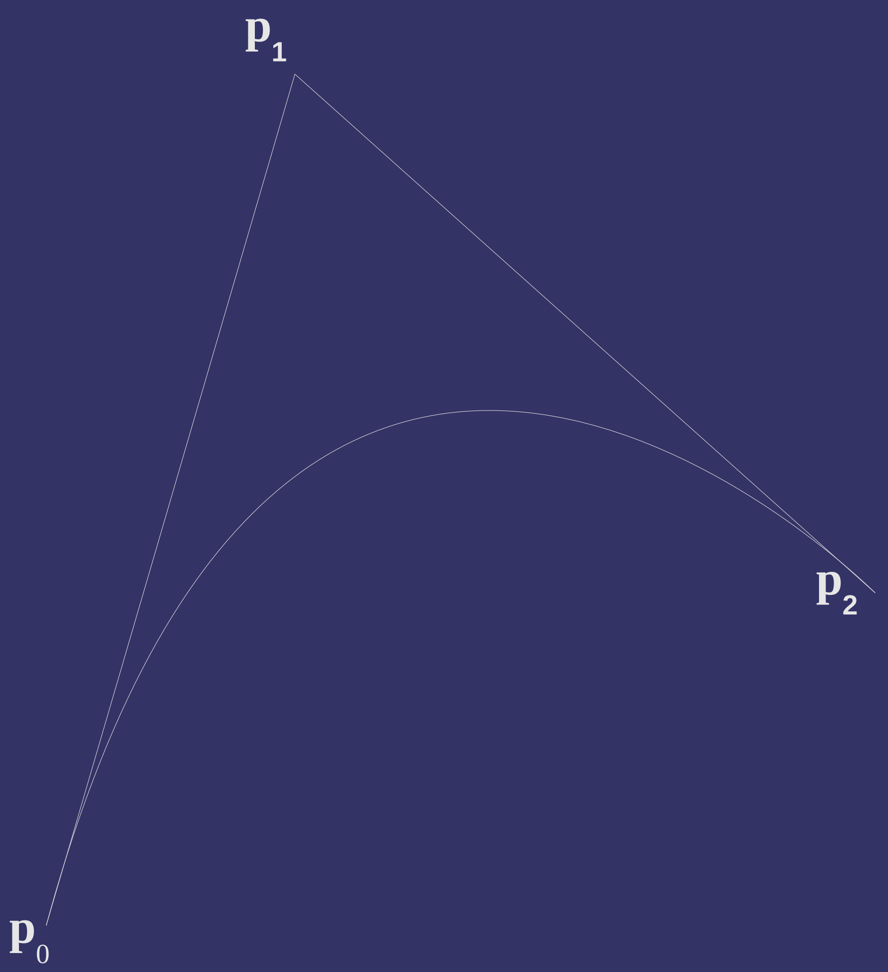
- We don't want the direction to suddenly change at the joint
  - Mathematically this means we want the function to be differentiable at the joint
  - This is the definition of C^1
- How is the derivative of a function at some point defined?

Piecewise Bézier Curves

- We don't want the direction to suddenly change at the joint
  - Mathematically this means we want the function to be differentiable at the joint
  - This is the definition of C^1
- How is the derivative of a function at some point defined?
  - It's the slope of a line *tangent* to the function at that point

Piecewise Bézier Curves

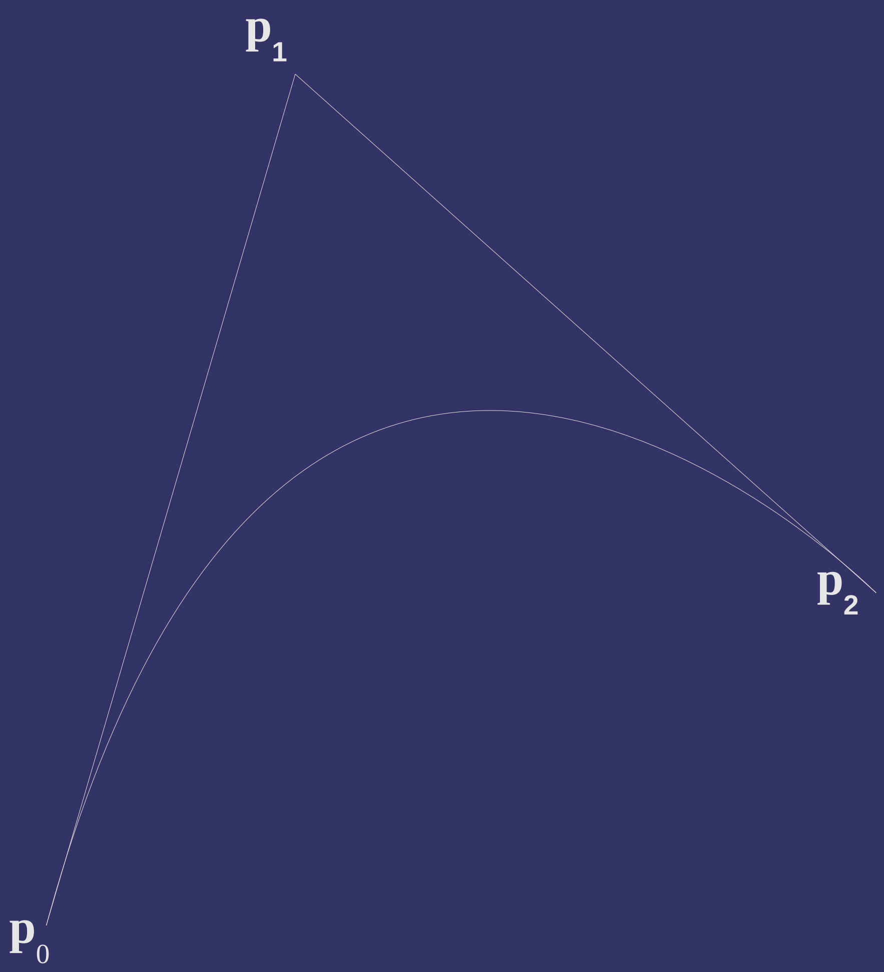
- What are the tangents at $\mathbf{p}_0$ and $\mathbf{p}_n$?



Piecewise Bézier Curves

- What are the tangents at $\mathbf{p}_0$ and $\mathbf{p}_n$?

$$\mathbf{m}_0 = \mathbf{p}_1 - \mathbf{p}_0$$
$$\mathbf{m}_1 = \mathbf{p}_n - \mathbf{p}_{n-1}$$



Piecewise Bézier Curves

- How can continuity be improved?
  - Let:
    - $\mathbf{m}_0$ = tangent at start of first curve
    - $\mathbf{m}_1$ = tangent at end of first curve
    - $\mathbf{m}_2$ = tangent at start of second curve
    - $\mathbf{m}_3$ = tangent at end of second curve
  - Modify $\mathbf{m}_1$ and $\mathbf{m}_2$ so that they are parallel

$$\frac{\mathbf{m}_1 \cdot \mathbf{m}_2}{|\mathbf{m}_1||\mathbf{m}_2|} = 1$$

Piecewise Bézier Curves

- If $|\mathbf{m}_1| \neq |\mathbf{m}_2|$ there will be a speed change at the joint
  - This is *not* C^1, but it's better than C^0
  - Sometimes G^1 for *geometrical continuity*

Derivative of a Bézier Curve

- Derivative using the sum rule and regrouping:

$$\frac{d}{dt}\mathbf{p}(t)=n\sum_{i=0}^{n-1}B_i^{n-1}(t)(\mathbf{p}_{i+1}-\mathbf{p}_i)$$

  - Exercise for the reader to confirm:

$$\frac{d}{dt}\mathbf{p}(0) = \mathbf{p}_1-\mathbf{p}_0$$

$$\frac{d}{dt}\mathbf{p}(1) = \mathbf{p}_n-\mathbf{p}_{n-1}$$

  - Result is a Bézier curve of one lower degree

Curved Surfaces

- Start with the same interpolation games
  - First extend from one parameter, t, to two parameters $\langle u, v \rangle$
  - Use four control points, $\mathbf{p}_{00}$, $\mathbf{p}_{01}$, $\mathbf{p}_{10}$, $\mathbf{p}_{11}$, instead of two
  - Interpolate between adjacent pairs:

$$\begin{aligned}
\mathbf{e} &= (1-u)\mathbf{p}_{00} + v\,\mathbf{p}_{01} \\
\mathbf{f} &= (1-u)\mathbf{p}_{10} + v\,\mathbf{p}_{11} \\
\mathbf{p}(u,v) &= (1-v)\mathbf{e} + v\,\mathbf{f} \\
&= (1-u)(1-v)\mathbf{p}_{00} + u(1-v)\mathbf{p}_{01} + (1-u)v\,\mathbf{p}_{10} + uv\,\mathbf{p}_{11}
\end{aligned}$$

  - Also known as *bilinear interpolation*

Curved Surfaces

- Extend to a curved surface in the same way as extending a line to a curve:
  - Add control points
    - For an $n \times m$ degree patch, there are $(n+1)(m+1)$ control points
    - Usually $n = m$
  - Recursively interpolate between the control points
    - Or use Bernstein form

Bézier Patches

- Bernstein form:

$$\mathbf{p}(u,v)=\sum_{i=0}^{m} \mathrm{B}_i^m(u) \sum_{j=0}^{n} \mathrm{B}_j^n(v) \mathbf{p}_{i,j}$$

  - As with Bézier curves:
    - Surface lies within convex hull of control points
    - And:

$$(u,v)\in[0,1]\times[0,1]\rightarrow B_i^m(u)B_j^n(v)\in[0,1]$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n} B_i^m(u)B_j^n(v)=1$$

    - Second summation is just a Bézier curve!

Bézier Patches

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Derivative of a Bézier Patch

- Similar to Bézier curves:

$$\frac{\partial \mathbf{p}(u,v)}{\partial u} = m\sum_{j=0}^{n}\sum_{i=0}^{m-1} B_i^{m-1}(u)\,B_j^{n}(v)\left[\mathbf{p}_{i+1,j}-\mathbf{p}_{i,j}\right]$$

$$\frac{\partial \mathbf{p}(u,v)}{\partial v} = n\sum_{i=0}^{m}\sum_{j=0}^{n-1} B_i^{m}(u)\,B_j^{n-1}(v)\left[\mathbf{p}_{i,j+1}-\mathbf{p}_{i,j}\right]$$

Normals of a Bézier Patch

- How do we calculate the normal?
  - What we *really* want is the normal of the plane tangent to the surface

Normals of a Bézier Patch

- How do we calculate the normal?
  - What we *really* want is the normal of the plane tangent to the surface
  - The partial derivatives give two vectors that lie in that plane... just take the cross product!

$$\mathbf{n}(u,v) = \frac{\partial \mathbf{p}(u,v)}{\partial u} \times \frac{\partial \mathbf{p}(u,v)}{\partial v}$$

Phong Shading Recap

- Phong shading... aka per-fragment lighting
  - Calculate lighting parameters per-vertex
  - Interpolate calculated values
  - Calculate lighting per-fragment based on interpolated parameter values

Phong Shading Recap

```
attribute vec3 normal;
attribute vec4 color;
uniform mat3 normal_xform;
uniform mat4 vertex_xform;
uniform mat4 mvp;

varying vec3 vertex_normal;
varying vec4 vertex_color;
varying vec3 vertex;

void main(void)
{
    gl_Position = mvp * gl_Vertex;

    vertex_normal = normal_xform * normal;
    vertex_color = color;
    vertex = vertex_xform * gl_Vertex;
}
```

Phong Shading Recap

```
uniform vec3 eye_space_light;
varying vec3 vertex_normal;
varying vec4 vertex_color;
varying vec3 vertex;
const vec3 eye_space_eye = vec3(0);

void main(void)
{
    vec3 l = normalize(eye_space_light – vertex);
    vec3 v = normalize(eye_space_eye - vertex);
    vec3 h = normalize(l + v);
    float n_dot_l = dot(vertex_normal, l);
    vec4 diff = vertex_color * n_dot_l;
    float spec = pow(dot(n, h), 16.0);

    gl_FragColor = step(0.0, n_dot_l) *
        vec4(diff.xyz + vec3(spec), vertex_color.w);
}
```

Surface-Space

- From the point of view of the surface, what is the normal vector?
  - We'll call this *surface-space*

Surface-Space

- From the point of view of the surface, what is the normal vector?
  - We'll call this *surface-space*
  - Assuming the surface is flat, $\mathbf{n}_{surf} = (0, 0, 1)$

Surface-Space

- If we know $\mathbf{n}_{world}$, can we create transformation that will generate $\mathbf{n}_{surf}$?
  - Not uniquely
    - An orthonormal basis requires three orthogonal, normalized vectors, but we only have one
      - If we have two we can generate the third
    - This is the same reason we need the “up” vector to create the camera look-at transform
  - If only we had another vector in plane...

Surface-Space

- Create a new vector, and call it the *tangent*
  - Either partial derivative of a Bézier patch can be used for $\mathbf{t}_{surf}$
    - Usually $\partial\mathbf{p}/\partial u$ is used
  - Knowing $\mathbf{n}_{surf}$ and $\mathbf{t}_{surf}$ is enough to create an orthonormal basis
  - This basis can transform *any* vector to surface-space from object-space
    - $\mathbf{n}_{obj}$ is an obvious choice
    - For lighting, $\mathbf{v}$ and $\mathbf{l}$ need to be in the same space as $\mathbf{n}$
- Because the tangent vector is used, surface-space is sometimes called *tangent-space*

Surface-Space

```
varying vec3 light_dir;
attribute vec3 tangent;
attribute vec3 normal;

void main(void)
{
    gl_Position = mvp * gl_Vertex;

    vec3 t = normal_xform * tangent;
    vec3 n = normal_xform * normal;
    mat3 tbn = mat3(t, n, cross(n, t));

    vec3 vert_pos = vec3(vertex_xform * gl_Vertex);
    vec3 light = eye_space_light - vert_pos;

    light_dir = normalize(light * tbn);
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Remember: $\mathbf{Mv} = \mathbf{vM}^T$

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Surface-Space

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varying vec3 light_dir;
varying vec3 eye_dir;
varying vec4 vertex_color;

void main(void)
{
    vec3 l = normalize(light_dir);
    vec3 v = normalize(eye_dir);
    vec3 h = normalize(l + v);
    float n_dot_l = l.z;
    vec4 diff = vertex_color * n_dot_l;
    float spec = pow(h.z, 16.0);

    gl_FragColor = step(0.0, n_dot_l) *
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}
```

Remember: **n** is (0, 0, 1)!

Surface-Space

- What is $\mathbf{b}$?
  - In the calculation: $\mathbf{b} = \mathbf{n} \times \mathbf{t}$
  - Correctly, this is the *bi-tangent*
    - Many places incorrectly call it the bi-normal
    - Either way, we'll just call it $\mathbf{b}$
  - Generally easier and more efficient to compute this in a shader than supply it as an input
    - We *cannot* just use $\partial\mathbf{p}/\partial v$ from from our surface evaluation because the two partial derivatives may not be orthogonal to each other!

Surface-Space

- What does this math headache gain us?
  - Just a trivial fragment shader optimization so far
    - Seems hardly worth it
  - What else?

Bump Mapping

- What if the surface isn't really flat or smoothly curved?
  - Just like few real surfaces have truly uniform color, few real surfaces have uniform normals
  - Use the same solution!
    - Store colors in an image → store normals in an image

Normal Map Storage

- Store the X, Y, and Z values of the surface-space normals in the R, G, and B components
  - Since Z tends to be close to 1.0, these images tend to look very blue

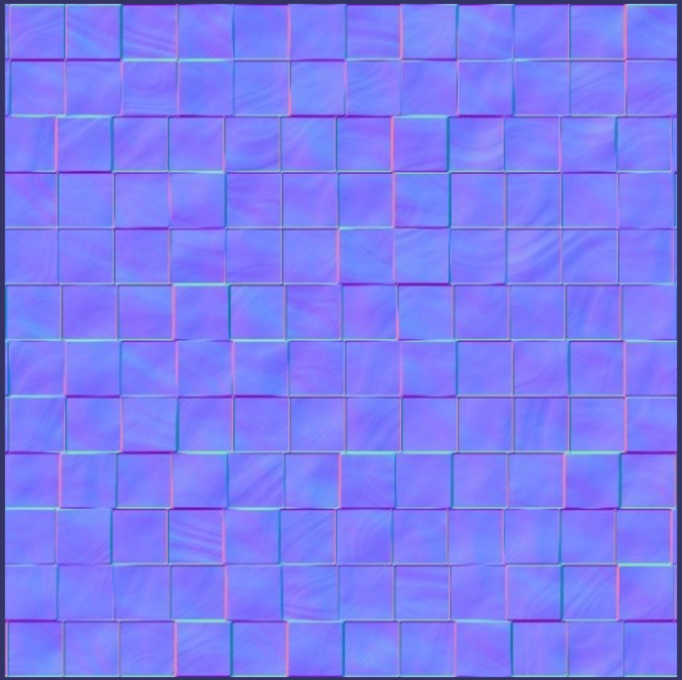

Image from http://www.filterforge.com/filters/243-normal.html

Normal Map Storage

- What is the range of colors in a texture?

Normal Map Storage

- What is the range of colors in a texture?
  - [0.0, 1.0]
  - We have to convert these to the [-1, 1] range desired for normal directions
    - Just convert X and Y... Z must be > 0, so just leave it

Normal Map Storage

- We don't even need Z
  - Z must always be > 0.0
  - Derive it from X and Y:

Normal Map Storage

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  - Z must always be > 0.0
  - Derive it from X and Y:

$$\sqrt{x^2+y^2+z^2}=1.0$$
$$x^2+y^2+z^2=1.0$$
$$z^2=1.0-x^2-y^2$$
$$z=\sqrt{1.0-x^2-y^2}$$

Normal Map Storage

- 2-component textures can be achieved in a couple ways:
  - Use `GL_LUMINANCE_ALPHA`
    - Some hardware doesn't really support this, so it will silently convert it to RGBA...making it bigger
  - Use `GL_RG`
    - Requires `GL_ARB_texture_rg` or OpenGL 3.0
  - Use `GL_COMPRESSED_RED_GREEN_RGTC2_EXT`
    - Requires `GL_ARB_texture_compression_rgtc`, `GL_EXT_texture_compression_rgtc`, or OpenGL 3.0
    - May add undesired compression artifacts

References

Lengyel, Eric. "Computing Tangent Space Basis Vectors for an Arbitrary Mesh". Terathon Software 3D Graphics Library, 2001. http://www.terathon.com/code/tangent.html

Normal map photography tutorial:

http://www.zarria.net/nrmphoto/nrmphoto.html

OpenGL extension specs:

http://www.opengl.org/registry/specs/ARB/texture_rg.txt

http://www.opengl.org/registry/specs/ARB/texture_compression_rgtc.txt

Next week...

- Render-to-texture
- Environment mapping
  - Rendering to env maps
- Improving the reflection model
  - Using env maps as better lights
  - Fresnel reflection
- Read:

Michael Toksvig. “Mipmapping Normal Maps.” http://developer.nvidia.com/object/mipmapping_normal_maps.html

Real-Time Rendering 3rd Edition, chapter 13.1 and 13.2.

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